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TECHNICAL REPORT R-126

ANALYSIS OF CLOSE LUNAR TRANSLATION TECHNIQUES

By JOSEPH N. SIVO, CARL E. CAMPBELL, and VLADIMIR HAMZA

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Lewis Research Center
Cleveland, Ohio

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By Joseph N. Sivo, Carl E. Campbell, and Vladimir Hamza

SUMMARY

An analysis of several techniques for close lunar *anslation was made to determine their relative* *ierits based primarily on fuel consumption. It* *as assumed that the lunar vehicle was in a near-* *overing condition at the start of the maneuver.* *ranslation was accomplished ballistically or at a* *onstant altitude. In the latter case, lateral thrust* *as obtained by tilting a single-engine vehicle or by* *ie use of auxiliary side-thrusting engines. The* *udy considered translations up to 5 statute miles in* *ngth.*

The ballistic maneuver consumed the least fuel *r a given range; however, the constant-altitude* *aneuvers were not greatly different. The single-* *ngine tilting vehicle was more efficient than the* *ultiengine system for constant-altitude translation.* *or constant-altitude translations of a given range,* *iel consumption decreased when the vehicle was* *llowed to coast (vertical thrust only) between the* *cceleration and deceleration portions of the flight* *s compared with no-coast translation.*

The total on-board fuel requirements of a lunar *inding vehicle decreased with decreasing hover* *ltitude regardless of the translational maneuvers* *onsidered.*

INTRODUCTION

With the advent of lunar expeditions using ianned vehicles, it appears that it will be neces- ıry to include in the vehicle design a capability f hovering near the lunar surface and/or of fecting horizontal translation prior to landing. ome of the reasons for this requirement include fined landing-site selection, lunar surface survey, ıd the need to reach a specific rendezvous point ı the lunar surface.

During the past 2 years, various studies of the utomatic landing phase from termination of the ring of the main retrorocket to the point of soft lunar contact have been conducted (e.g., refs. 1 and 2). The primary emphasis in these studies has been on guidance instrumentation and propulsion system requirements which will satisfy the objective of a soft landing. Some thought has been given to the alteration of the descent trajectory in order to accomplish a translation of the impact point. It is generally agreed that, if an alteration of the flight path is to be made to change the impact point, the sooner it is accomplished, the more efficient will be the fuel consumption during the descent. However, it is quite possible that a final decision will be required in very close proximity to the lunar surface, owing to problems of visibility and resolution through the rocket exhaust and possibly lunar dust and to vehicle vibration effects on the observer.

Related herein is a brief analysis of several types of translational maneuvers that can be used after the vicinity of the lunar surface has been reached and the vehicle brought to a hover or near-hover condition. The initial conditions of the terminal maneuver can be reached by retrorocket firing during a collision trajectory approach or a grazing trajectory approach to the lunar surface. The types of translation maneuvers presented are constant-altitude translation using both single and multiengine systems and a ballistic translation which includes the final phase of descent to the lunar surface. These maneuvers were analyzed over a range of initial altitudes up to 40,000 feet for thrust-to-lunar-weight ratios up to 10 and for translation distances up to 5 statute miles. The relative merits of each type of maneuver are discussed mainly on the basis of fuel consumption for propulsion systems with both variable- and constant-thrust engines. The time factor associated with the various maneuvers is also discussed.

Charts for the solutions of the motion equations for ballistic-type translational maneuvers are presented in appendix C. These charts provide a relatively rapid means of approximating the requirements of ballistic translation for a variety of initial conditions.

ANALYSIS OF BASIC MANEUVERS

The following analysis assumes the vehicle to be a point mass concentrated at the vehicle center of gravity and maneuvering in a constant-gravity field. All weights presented are moon weights. In all cases, the horizontal and vertical velocity components at the end of the translational maneuver are zero. The following analysis is presented in more detail in appendix B.

BALLISTIC TRANSLATION

A sketch of typical vehicle trajectories during a ballistic maneuver is shown in figure 1. The general procedure in performing this maneuver would begin with rotation of the vehicle to the proper firing angle. Thrust is then initiated (point A) and maintained while the firing angle (thrust vector angle) is held constant with respect to the local vertical. A constant firing angle, which is optimized to maximize the range during boost and the free-flight phase, is used to facilitate the calculations. Although the use of a fixed firing angle is not necessarily a true optimum procedure for the burning times involved, the non-optimum effects can be considered negligible. The thrust is terminated and the vehicle is allowed

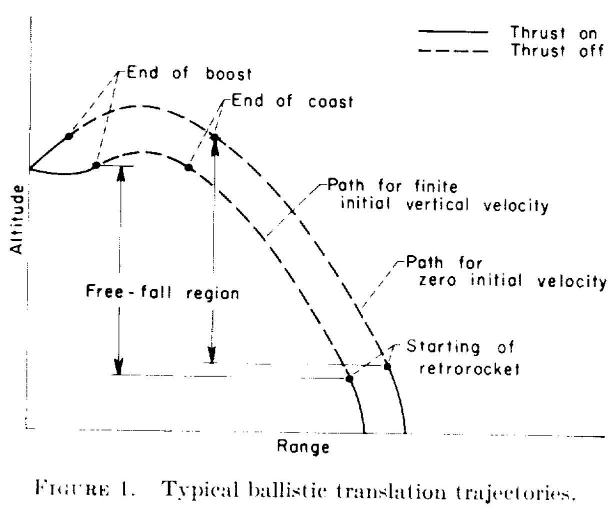


FIGURE 1. Typical ballistic translation trajectories.

to coast and fall freely to the retrorocket firir altitude. Prior to the retrorocket phase, tl vehicle is alined along the velocity vector. Tl thrust vector is maintained along the velocit vector during retrorocket firing until zero velocit is reached. Thrusting along the velocity vect ensures simultaneous termination of both horizor tal and vertical velocity components.

In the ballistic maneuver, with constant thrus weight ratio maintained during burning (variabl thrust engine), the total range is made up of boost phase, a free-flight or unpowered phase, an a retrothrust phase. During the boost phas range change occurs as a result of initial horizont velocity (ΔR_0) and boost thrust (ΔR_b). Durin free flight, the range change results from a coas to maximum altitude and return to burnout alt tude when the burnout velocity is directed upwar (ΔR_c), followed by a free fall from burnout altitud to the retrothrust initial altitude (ΔR_{ff}). Rang during boost and free-flight phases was obtaine with closed-form solutions of the motion equation since time is the only variable. However, durin retrothrust, the range ΔR_r was obtained by a analog solution of the motion equations in whic both magnitude and direction of the velocity a variables.

The fuel consumption during the maneuver given by the following equation:

$$\left(\frac{w_f}{w_0}\right)_t = 1 - exp\left[-\frac{(F_0/w_0)(t_b+t_r)}{aI_{sp}}\right]$$

(Symbols are defined in appendix A.)

A graphical solution of the ballistic maneuv with constant thrust-weight ratio is given i appendix C.

The equations that result if a constant-thrus engine is used are much the same as with constar thrust-weight ratio. In this instance to simplif the calculations the range during retrothrust ΔI was determined from a closed-form solution of tl motion equations made possible by assuming th the thrust vector angle was constant with respe to the local vertical during retrothrust. This sti requires an iterative solution to obtain zer velocity components at the termination altitud however, since the constant-thrust case was onl used as a point of comparison, a machine comp tation was not warranted.

The resulting fuel consumptions during boost nd retrothrust are as follows:

$$\left(\frac{w_f}{w_0}\right)_b=\frac{(F/w_0)t_b}{aI_{sp}}$$

$$\left(\frac{w_f}{w_0}\right)_r=\frac{(F/w_0)t_r}{aI_{sp}}$$

HORIZONTAL TRANSLATION

This maneuver can be performed with a vehicle vhich provides separate engines for the vertical nd horizontal thrust requirements or with a tilting 'ehicle that uses a single engine to provide both hrust components.

With the multiengine system, the procedure vould be to accelerate horizontally from a hovering condition with the horizontally mounted engine o some velocity, then coast with this engine off, vith altitude being sustained by the vertically iring engine. The vehicle would then be decelertted to a hovering condition with either a dianetrically opposed horizontal engine or after a 'ehicle rotation of 180° with the same horizontal ngine.

The total range of the translation using the nultiengine system is given by the following quation:

$$R_t=\Delta R_{accel}+\Delta R_c+\Delta R_{decel}$$

vhere $\Delta R_{accel}=\Delta R_{decel}$ when the horizontal engine naintains a constant thrust-weight ratio.

The resulting fuel consumption for this system s given by

$$\left(\frac{w_f}{w_0}\right)_t=1-exp\left\{-\frac{2[1+(F/w)_b]t_b}{aI_{sp}}-\frac{t_c}{aI_{sp}}\right\}$$

The single-engine system requires that the vehicle be tilted to a prescribed angle to provide horizontal acceleration. This necessitates an increase in engine thrust during tilting in order to naintain a vertical thrust component equal to the 'ehicle weight. During the tilting, a horizontal ranslation occurs. The vehicle then translates at constant altitude and constant thrust angle vhere the engine thrust-weight ratio is presumed onstant. The vehicle is then returned to the 'ertical position with thrust equal to weight and llowed to coast. The tilting procedure is dupliated in the reverse direction to decelerate the vehicle. A detailed discussion of firing angle optimization is presented later in the report.

The total range equation using a single engine is somewhat more complicated than that for the multiengine case, since translation occurs during vehicle tilting. The equation is

$$R_t=\Delta R\Big]_0^{\varphi} tilt+\Delta R_{accel}+\Delta R\Big]_{\varphi}^{0} tilt+\Delta R_c$$
$$+\Delta R\Big]_0^{-\varphi} tilt+\Delta R_{decel}+\Delta R\Big]_{-\varphi}^{0} tilt$$

where

$$\Delta R\Big]_0^{\varphi}+\Delta R\Big]_{\varphi}^{0}+\Delta R\Big]_0^{-\varphi}+\Delta R\Big]_{-\varphi}^{0}$$

denotes the range covered during the vehicle orientations required to provide horizontal thrust and φ is the thrust angle relative to the local vertical.

The fuel consumption is also somewhat involved and is as follows:

$$\left(\frac{w_f}{w_0}\right)_t=1-\left[(\sec\varphi+\tan\varphi)^{-\frac{4}{\omega aI_{sp}}}\right] exp\left[-\frac{2(F/w_0)t_b}{aI_{sp}}\right] exp\left[-\frac{t_c}{aI_{sp}}\right]$$

DISCUSSION

It was considered beyond the scope of this analysis to treat the many thrust modulation techniques possible for lunar translation. The most obvious approaches and the most amenable to analysis are to use either constant thrust or thrust which is varied to hold a constant thrust-weight ratio. Of the two methods, constant thrust-weight ratio is the simpler to compute and is primarily used in this report as the thrust control technique. As might be expected, this technique will not exactly assess the fuel required for all thrust modulation techniques, but it will yield numbers that can be considered typical. To illustrate this, a comparison of fuel costs is made between a constant-thrust system and a constant-thrust-weight-ratio system for a vertical descent maneuver and a ballistic translational maneuver in figures 2 and 3, respectively. Free falls from 20,000 and 10,000 feet were arbitarily chosen to illustrate the vertical descent maneuver (fig. 2). The lowest values of fuel-to-initial-weight ratio occur with the constant-thrust techniques as

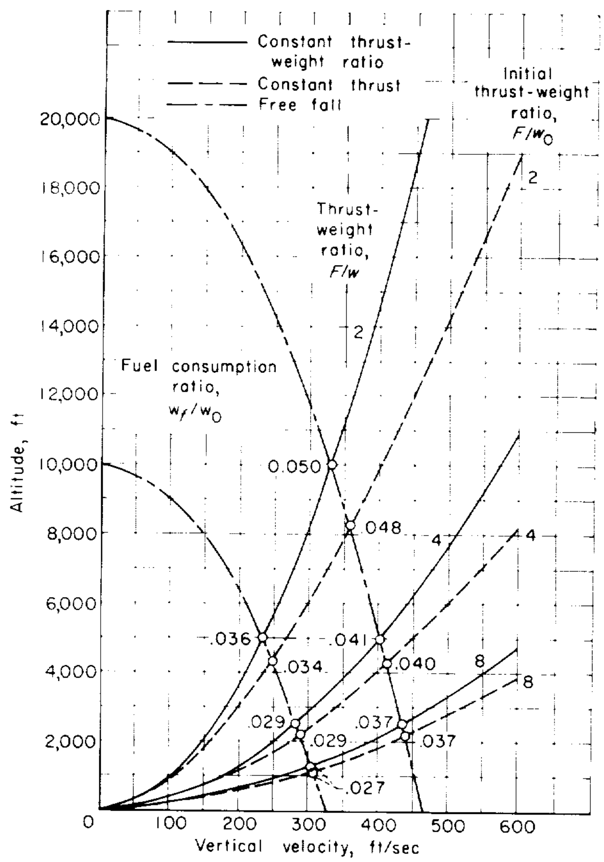


FIGURE 2.—Fuel cost comparison of constant-thrust and constant-thrust-weight-ratio systems for vertical descent.

shown by the values noted at comparable thrust initiative points. However, at thrust-weight ratios greater than 4, the difference in the two thrust modulation methods can be considered negligible. Here and in all subsequent figures and examples, the specific impulse has a value of 400 seconds.

In the ballistic translation maneuver selected for this example (fig. 3), the initial altitude is equal to the final altitude (see sketch in fig. 3). The firing angle is held constant during boost at a value which maximizes the range and minimizes the fuel consumption during the boost-free flight phase of the flight. The optimum constant firing angle φ as a function of the thrust-weight ratio is presented in figure 4. The optimum, which includes the case where retrothrust initiation occurs below the initial altitude, is obtained by partial differentiation of the range equation for the boost-free flight phase as a function of the angle φ for constant thrust-weight ratio. Although the optimum angl was obtained using constant thrust-weight ratio, i can be and is used for the constant-thrust case wit little error. During the retro thrust portion, th thrust vector is maintained along the velocit vector until zero velocity is attained. Again th constant-thrust system yields the lowest fu ratios; however, as before, the difference is quit small and can be considered negligible in the rang of interest, that is, translation distances up to statute miles.

It is apparent that the small difference betwee constant thrust and constant thrust-weight rati for the ballistic case will not alter the trends tha will be established using the constant-thrust weight-ratio technique. It is shown later tha this is also the case when considering constant altitude translation techniques.

The discussion to follow considers three maneu vers, ballistic translation, constant-altitude trans lation with a single-engine system, and constant altitude translation with a multiengine system Typical cases of translational maneuvers wer

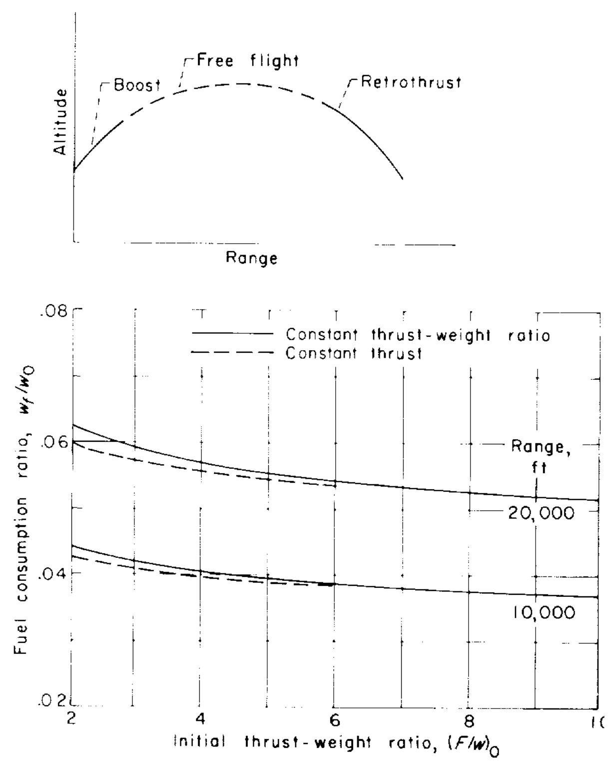


FIGURE 3.—Fuel cost comparison of constant-thrust an constant-thrust-weight-ratio systems for ballistic trans lation.

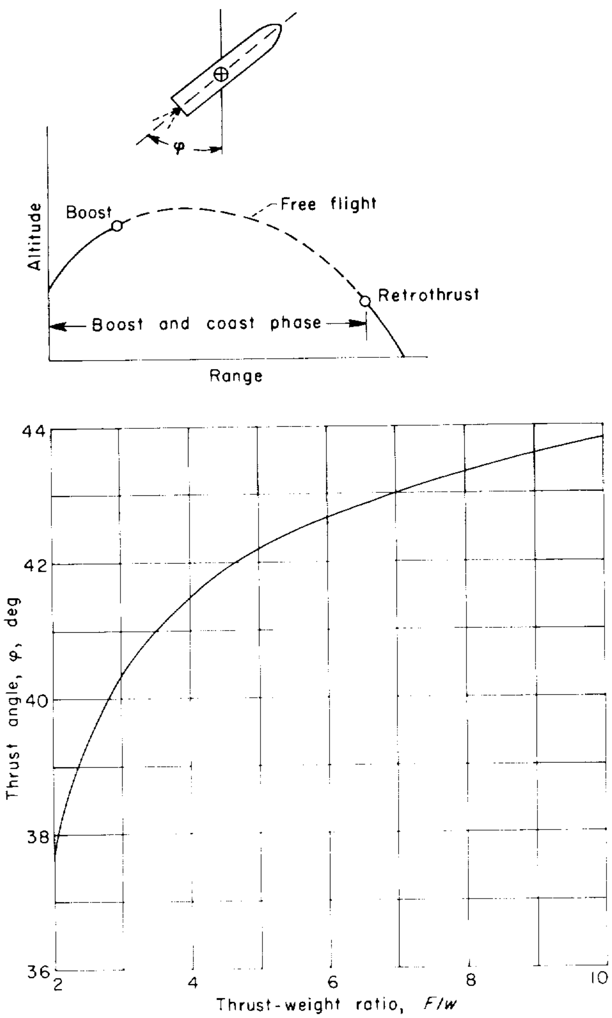


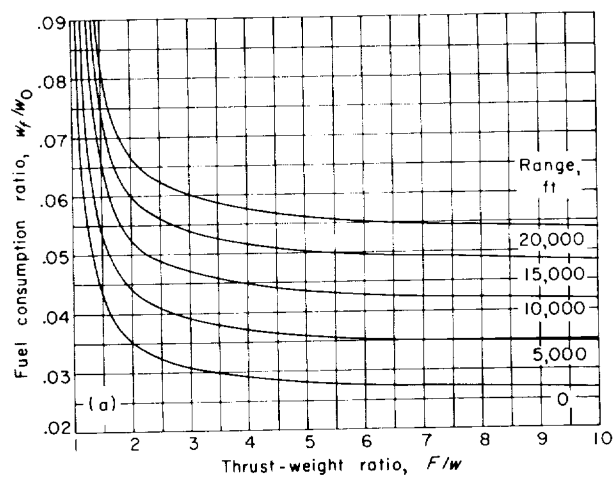
FIGURE 4.—Optimum thrust angle for given constant thrust-weight ratio to maximize boost and coast phase.

selected to illustrate the relative merits of each maneuver based on the theoretical fuel consumption and maneuver time. Each maneuver is first discussed individually to illustrate the effect of system variables that pertain directly to the maneuver under consideration. A relative comparison of the three types is then made.

BALLISTIC TRANSLATION

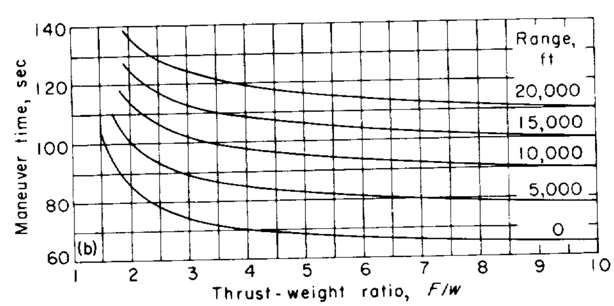
The effect of engine thrust-weight ratio on overall fuel consumption (retrothrust and boost thrust fuel) and maneuver time for the ballistic maneuver is shown in figure 5. An initial altitude of 10,000 feet with an initial velocity of zero is assumed, with retrothrust termination at zero altitude and velocity. The firing angle during boost is held constant at an angle which maximizes range during the boost-coast phase of the flight. During retrothrust, the thrust vector is maintained along the velocity vector. The rotation time required to aline the vehicle prior to boost is assumed to be zero.

Increasing the thrust-weight ratio above 4 (fig. 5(a)) results in only a relatively small reduction in fuel consumption for a given range. It should be noted that a large portion of the total fuel consumed in this case is used to overcome the vehicle's initial potential energy, as indicated by the zero-range curve. The percentage cost of fuel per unit horizontal distance traveled decreases



(a) Fuel consumption.

FIGURE 5.—Effect of thrust-weight ratio on fuel consumption and maneuver time for ballistic translation. Initial altitude, 10,000 feet; initial velocity, 0.



(b) Maneuver time.

FIGURE 5.—Concluded. Effect of thrust-weight ratio on fuel consumption and maneuver time for ballistic translation. Initial altitude, 10,000 feet; initial velocity, 0.

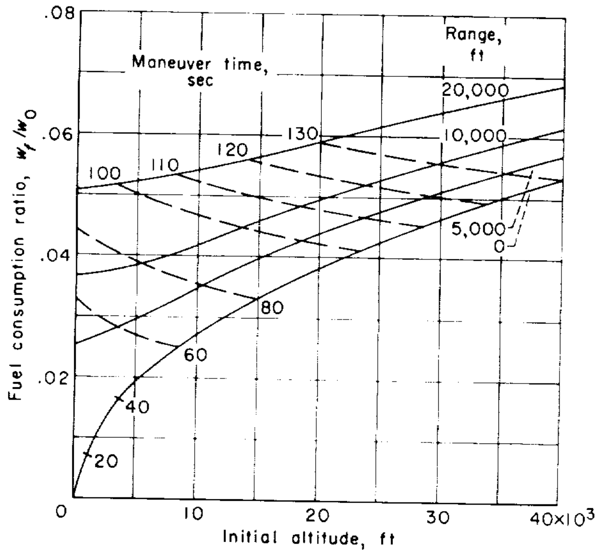


FIGURE 6. Effect of initial altitude on overall fuel consumption. Ballistic translation during descent for initial velocity of zero and thrust-weight ratio of 8.

slightly with increasing range for a given thrust-weight ratio.

The duration of the maneuver which includes boost, coast, and retrothrust times decreases with increasing thrust-weight ratio (fig. 5(b)). However, only a 20-percent reduction occurs when the thrust-weight ratio increases from 2 to 10. The approximate time available for observation and decision during the coast portion of the maneuver is 70 to 110 seconds for the translation ranges being considered.

The effect of the initial hover altitude on the overall fuel consumption of the boost plus retrothrust system is shown in figure 6 for a thrust-weight ratio of 8. Also indicated in the figure are the approximate maneuver times involved. The same assumptions apply here as in figure 5. As can be seen by the range curves, the lower the hover altitude, the lower the fuel consumption for a given translation (including the fuel required to bring the vehicle to zero altitude). Again, as noted in connection with figure 5, a large portion of the fuel required for this terminal maneuver is expended to overcome the initial potential energy of the vehicle (initial hover altitude), as indicated by the zero-range curve.

If the main retrothrust system which delivered the vehicle to the hover altitude is considered as it should be when assessing the overall mission fuel consumption, the lowest hover altitude commensurate with the requirements of the overall mission is still the most desirable. This is the case because a choice of hover altitudes is available without requiring a significant change in the main retrothrust system fuel requirements. This is essentially true for all descent trajectories. However, it can most easily be seen by examining the characteristic velocity increment ΔV of the main retrothrust system for a vertical descent trajectory, which is given by

$$\Delta V = g_e I_{sp} \ln \frac{m_r}{m_0} + \int_0^{t_b} g \, dt$$

For a practical lunar descent, the velocity increment given by the "gravity term" $\int_0^t g\,dt$ is small relative to the total velocity increment, amounting to approximately 10 percent. If the altitude at which the main retrorocket is fired were raised, while all other quantities remained fixed, the total velocity increment would change only as a result of the small change in the gravity field experienced by the vehicle during retrothrust, which affects only the gravity term, and a small change in approach velocity. The change in the hover altitude would then be essentially equal to the change in the initial retrothrust altitude, and the fuel requirement would be unchanged.

After the vehicle has been delivered to a hover condition, the maneuver which combines simultaneously a vertical descent with a horizontal translation will require the lowest fuel consumption to attain a given range. This can be clearly seen from figure 6. A vertical descent from 40,000 feet to the lunar surface requires 5.27 percent of the vehicle weight at hover. A subsequent 10,000-foot translation requires 3.58 percent of the remaining vehicle weight or a total of 8.66 percent of the original weight to descend from 40,000 feet and translate 10,000 feet. If these separate maneuvers were performed simultaneously, the total cost would be only 6.18 percent of the original vehicle weight.

Until now, the vehicle has been assumed to start the terminal maneuver from a hover condition. Consideration will now be given to the case where the vehicle has an initial downward vertical velocity at the start of the maneuver. Figure 7 shows the effect of initial velocity on fuel consump-

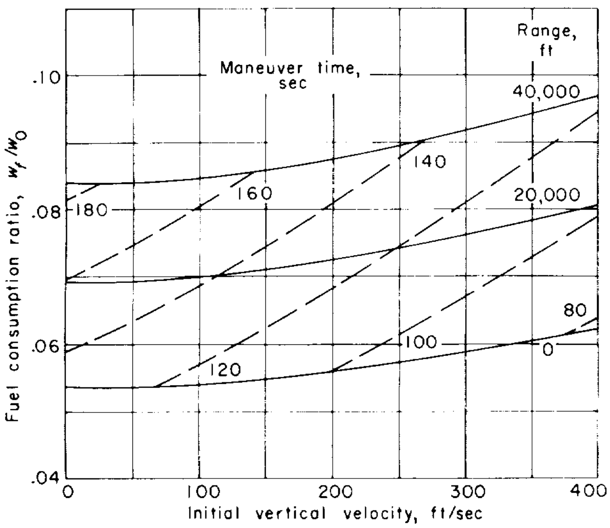


FIGURE 7.—Effect of initial vertical velocity on fuel consumption and maneuver time. Ballistic translation during descent for initial altitude of 40,000 feet and thrust-weight ratio of 8.

tion for a combined descent and translation from 40,000 feet. The fuel ratio presented is based on the initial weight of the vehicle for the 40,000-foot altitude and the initial velocity in question. Under these provisions the fuel consumption for the maneuver increases with increasing velocity. At an initial altitude of 40,000 feet an increase in vertical velocity from 0 to 400 feet per second increases the vehicle energy level approximately 18 percent. As expected, this results in a comparable increase in fuel consumption. The percentage of the total energy of the vehicle represented by the initial vertical velocity varies inversely with the initial altitude. Therefore, the slope of fuel consumption as a function of initial velocity for a given range also varies inversely with altitude. Maneuver time is also affected by initial velocity and is reduced approximately 35 to 40 percent when the velocity is increased from 0 to 400 feet per second.

Just as the effect of hover altitude on overall mission requirements has been discussed, so also the overall effect of initial vertical velocity on overall mission requirements must be evaluated. To illustrate the fuel requirements for a typical main retrothrust system, figure 8 relates the fuel requirements to the velocity change accomplished during retrothrust for a vertical descent maneuver. Consider, for example, a typical mission with a velocity change of 9000 feet per second. To reach a hover altitude a decrease in velocity change of 400 feet per second (burnout velocity of 400 ft/sec) increases the burnout weight 1.6 percent. This saving can then be credited to any subsequent translation maneuver. Assuming that the velocity decrease of 9000 feet per second brings the vehicle to a 40,000-foot hover altitude, the main retrothrust system requires 52.8 percent of the system weight (fig. 8); the descent from 40,000 feet, including a 20,000-foot translation, then requires 6.9 percent of the remaining weight (fig. 7). This is a total of 56.1 percent of the original weight. If, however, an initial vertical velocity of 400 feet per second is allowed at the start of the final descent translation maneuver, the main retrothrust

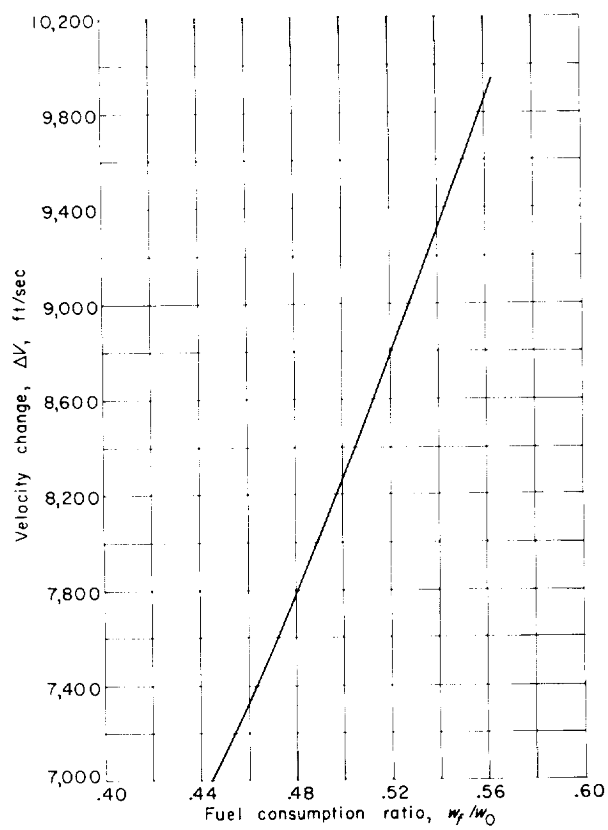


FIGURE 8.—Effect of main retrothrust system velocity change requirements on fuel consumption ratio for constant-thrust engine. Specific impulse, 400 seconds; thrust-mass ratio, 50; average acceleration due to gravity, 5.0 feet per second per second. $\Delta V = g_e I_{sp} \ln \left[1 - \frac{(F/w_0)t_b}{a I_{sp}} \right] + \bar{g} t_b$; $w_f/w_0 = \frac{(F/w_0)t_b}{a I_{sp}}$.

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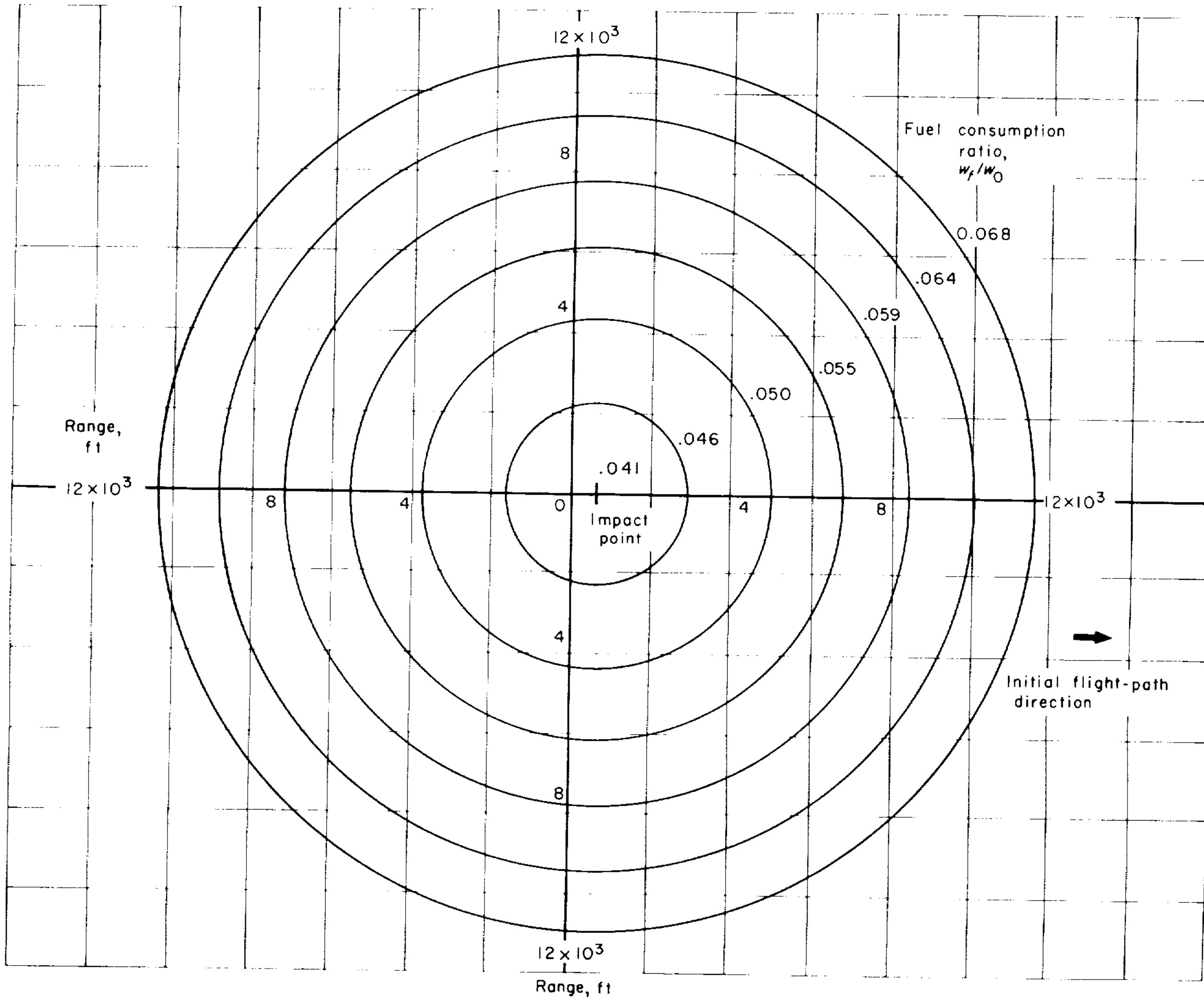
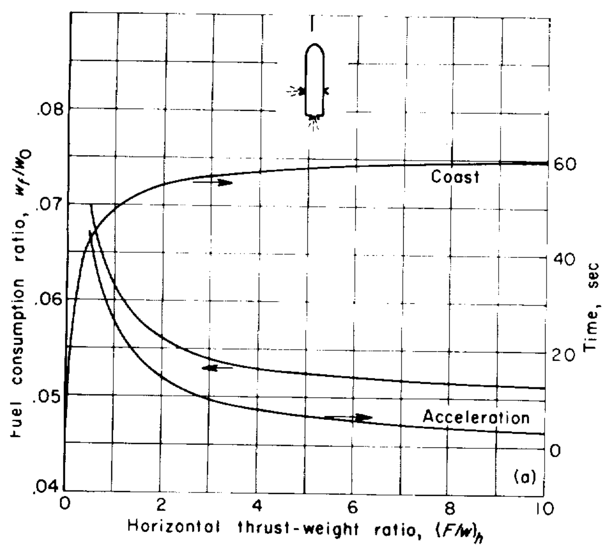


FIGURE 9.—Combined effect of initial vertical and horizontal velocity components on range and fuel consumption. Initial conditions: horizontal velocity, 20 feet per second; vertical velocity, 200 feet per second; altitude, 10,000 feet; thrust-weight ratio, 2.0.

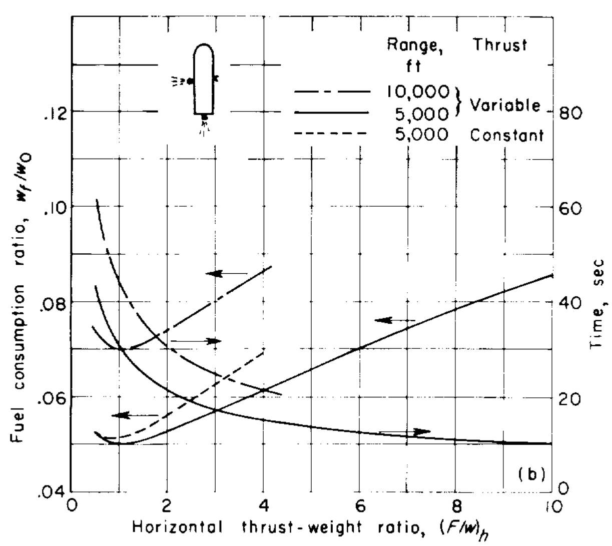
system requires 51.3 percent of the original weight and the final maneuver 8.1 percent of the remaining weight. The total is then 55.2 percent or a net saving of 0.9 percent of the original vehicle weight, which may represent a significant fraction of the useful payload. The general conclusion concerning selection of the optimum hovering altitude and vertical velocity distribution between main and final retrothrust is as follows: The lower the hovering altitude and the higher the velocity at the end of main retrothrust, the lower will be the fuel cost for the overall mission. However, in addition to considerations of overall fuel requirement, the selection of hovering conditions for a specific vehicle will also be governed by other factors such as the final retrothrust engine thrust capability, sensor and control accuracies, lunar surface irregularities, and so forth.

The effect of a nonvertical initial trajectory at initiation of a ballistic translation maneuver on range capabilities in any direction is shown in figure 9. As would be expected, the fuel cost for translating a given distance from the initiation point is a minimum in the direction of the initial horizontal velocity and maximum in the opposite direction. However, the main point of interest is that the contours of constant fuel consumption are essentially circular with their centers at the impact point, which is defined as the landing point that would be reached if no attempt were made to translate. This concentric circle relation between the constant-fuel-consumption contours and the impact point is a good approximation with horizontal velocity components to 100 feet per second.



(a) Translation with coast. Range, 10,000 feet.

FIGURE 10.—Effect of horizontal engine thrust-weight ratio on fuel consumption and maneuver time for multiengine system.



(b) Translation without coast.

FIGURE 10.—Concluded. Effect of horizontal engine thrust-weight ratio on fuel consumption and maneuver time for multiengine system.

It can be concluded that, for short ranges, the fuel cost for translating a given distance from the no-translation impact point is independent of translation direction.

CONSTANT-ALTITUDE TRANSLATION

Multiengine system.—The first propulsion system considered is the multiengine system which has separate engines to provide (1) horizontal vehicle acceleration and (2) vertical thrust to hold altitude constant. The effect of the horizontal engine thrust-weight ratio (ratio held constant during maneuver) on fuel consumption for translation only for both vertical and horizontal engines and on maneuver time is shown in figure 10 for a range of 10,000 feet.

Two general maneuver techniques are employed in the analysis. One allows a vehicle coast period at constant horizontal velocity between the acceleration and deceleration phases of the maneuver. The other has continuous horizontal thrust application and thereby zero coast time. In figure 10(a), variations in fuel consumption during translation, coast time, and acceleration time (equal to deceleration time) are shown as a function of the horizontal engine thrust-weight ratio for the maneuver using a coast period. The total maneuver time is equal to twice the acceleration time plus the coast time. The relation between thrust-weight ratio and coast time as shown is that which results in minimum fuel consumption and is given by

$$t_c = 2\sqrt{R/g}\sqrt{\frac{(F/w)_h}{1+2(F/w)_h}}$$

(see appendix B). Fuel consumption decreases with increasing thrust-weight ratio; however, only small additional reduction occurs beyond a value of 4. Since the time with the engine on (acceleration time) decreases very rapidly to a very few seconds with increasing thrust-weight ratio, the vehicle and engine control response time requirements as well as engine weight limitations will limit maximum practical thrust-weight ratio.

When the coast period is eliminated, the effect of thrust-weight ratio on fuel consumption is somewhat different, as shown in figure 10(b) for ranges of 5000 and 10,000 feet with variable thrust (constant thrust-weight ratio) and for a range of 5000 feet with constant thrust. An optimum thrust-weight ratio is evident at a value of 1.0

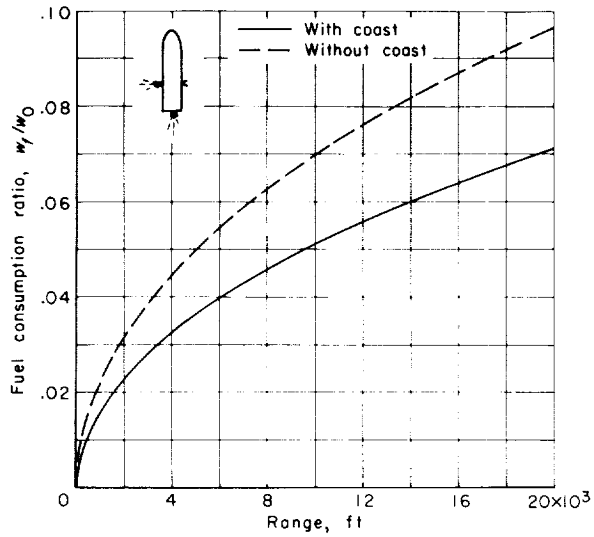
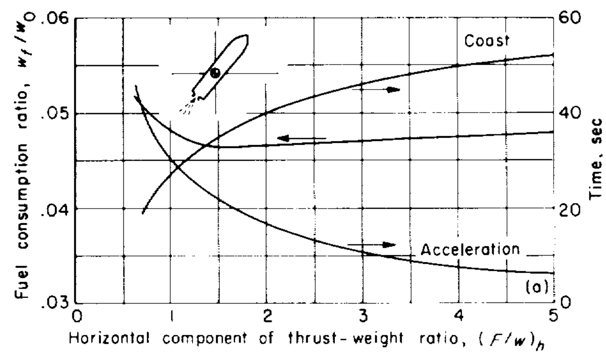


FIGURE 11.—Comparison of fuel cost with and without coast for horizontal (constant-altitude) translation using multiple engines.

and is independent of range when variable thrust is used. If a constant-thrust horizontal engine were used, its performance as a function of its initial thrust-weight ratio would be as shown by the dashed line for the 5000-foot range. A constant-thrust engine cannot maintain the optimum value during the translation because of decreasing vehicle mass. Therefore, as shown, it will not be as efficient as the engine system which can maintain the optimum constant thrust-weight ratio of 1.0.

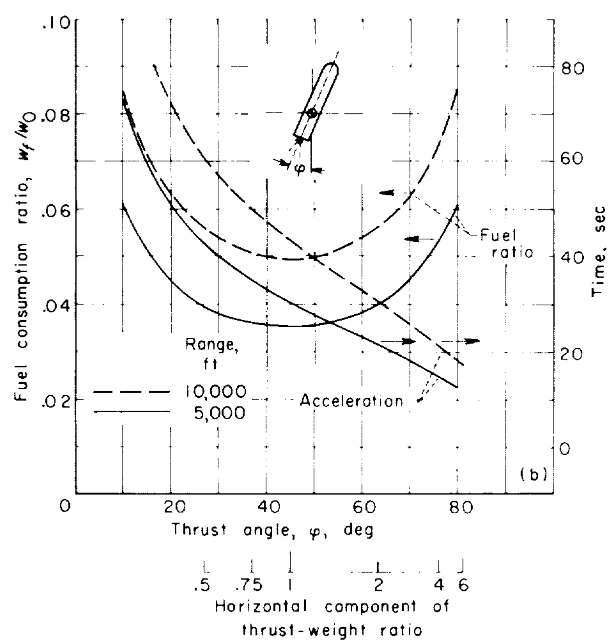
A comparison between maneuvers with and without coast is illustrated in figure 11, where fuel consumption is presented as a function of range. The maneuver with coast is the more efficient. For example, with the 10,000-foot range, the fuel consumption ratio is 0.07 for zero coast and 0.05 with coast. A horizontal thrust-weight ratio of 10 was used in the case with coast to illustrate the near-minimum fuel consumptions; however, as mentioned, engine weight considerations and engine dynamic characteristics might greatly limit the maximum allowable thrust-weight ratio. The optimum thrust-weight ratio of 1.0 was used in the zero-coast case. In general, the fuel cost per unit distance decreases with increasing range.

Single-engine system.—If the constant-altitude translation is performed using a single-engine system, thus requiring vehicle tilt, the effect of the horizontal component of the engine thrust-weight ratio on fuel consumption and maneuver time i as shown in figure 12. The horizontal componen of thrust is used as the abscissa to allow a mor direct comparison of performance with the multi engine system. As with the multiengine system the maneuver can be performed with and withou a coast period between acceleration and decelera tion of the vehicle. The variables of fuel con sumption, coast time, and acceleration time ar



(a) Translation with coast. Range, 10,000 feet.

FIGURE 12.—Effect of horizontal component of thrust weight ratio on fuel consumption and maneuver time fo single-engine system.



(b) Translation without coast.

FIGURE 12.—Concluded. Effect of horizontal componen of thrust-weight ratio on fuel consumption and maneuve time for single-engine system.

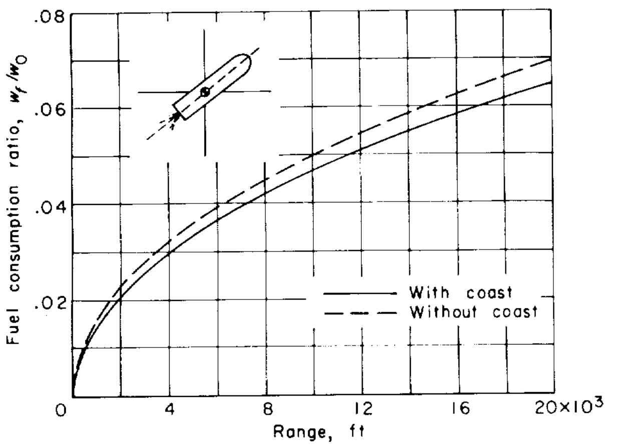


FIGURE 13.—Comparison of fuel consumption with and without coast for horizontal (constant-altitude) translation using single engine.

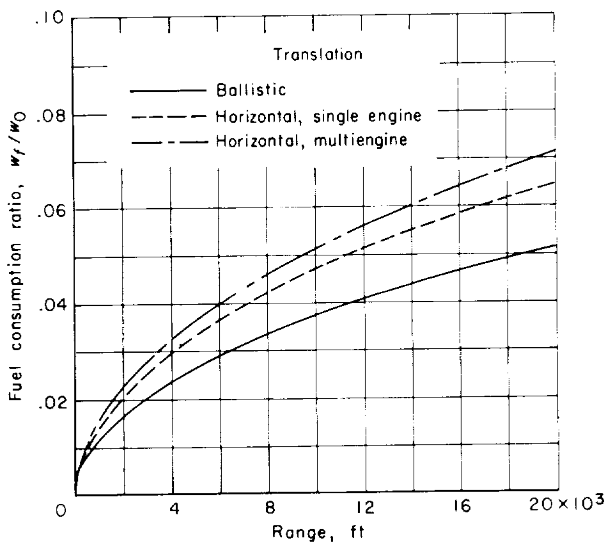


FIGURE 14. Fuel cost comparison of three translation techniques.

shown in figure 12(a) as functions of the horizontal component of engine thrust-weight ratio for the maneuver with coast. The relation between engine thrust-weight ratio and coast time shown is that which results in minimum fuel consumption and is given by

$$t_c=2\sqrt{R/g}\,\frac{(F/w)-1}{\sqrt{[2(F/w)-1][(F/w)^2-1]^{1/2}}}$$

(see appendix B).

The minimum fuel consumption for any range is obtained with an engine thrust-weight ratio of 2, which corresponds to a horizontal-component thrust-weight ratio of $\sqrt{3}$ and a tilt angle measured from the vertical of 60°. For thrust-weight ratios greater than optimum, the fuel ratio is relatively insensitive to changes in thrust. However, total maneuver time (acceleration, coast, deceleration) is reduced as thrust-weight ratio is increased.

If the coast period is eliminated, the optimum engine thrust-weight ratio decreases to $\sqrt{2}$. As shown in figure 12(b), this corresponds to an optimum horizontal $(F/w)_h$ of 1.0 and a thrust angle of 45° and is independent of the range. The fuel ratio is relatively insensitive to thrust-angle change near the optimum of 45°.

In figure 12 it was assumed that the vehicle rotation time was zero. An analysis of the effect of vehicle rotation rate on fuel consumption was made, and the results indicate no significant increase in fuel consumption occurs for rotation rates of practical interest.

The effect of range on fuel consumption using a single-engine system with and without a coast period is shown in figure 13. The maneuver with coast is the more efficient of the two, as is the case with multiengine systems; however, the potential gain is less with a 0.05 fuel ratio without coast and a 0.047 fuel ratio with coast at the 10,000-foot range. The optimum value of thrust-weight ratio is used in each case. In general, as in the multiengine case, the fuel cost per unit distance traveled decreases with increasing range.

COMPARISON OF MANEUVERS

A comparison of the three translation techniques (ballistic, constant-altitude with a multiengine system, and constant-altitude with a single-engine system) on the basis of overall fuel costs is made in figure 14. It was assumed that the initial altitude and velocity of the vehicle were zero, and for constant-altitude translation an optimum coast period was used. Because of obvious practical considerations, the engine thrust-weight ratio was limited to a maximum of 10 for the ballistic and the multiengine constant-altitude cases, although the optimum value of thrust-weight ratio is infinity. The time required for vehicle orientation was assumed to be zero in all cases. The ballistic maneuver required the lowest fuel consumption for a given range. The fuel ratios for the 20,000-foot range for the ballistic case, constant-altitude single-engine case, and constant-altitude multiengine case were 0.052, 0.065, and 0.071, respectively.

SUMMARY OF RESULTS

The present analysis concerned mainly the mode of translation starting from a hovering position but also included a method for analysis with initial horizontal and vertical velocities. Some specific results obtained in this analysis are:

1. A translational maneuver performed from a hover position at essentially zero altitude can result in a significant fuel consumption. For instance, 5 to 6 percent of the vehicle weight may be consumed while translating 20,000 feet horizontally (approximately 2½ percent of the vehicle weight approaching the moon).

2. The greater the initial hovering altitude, the higher will be the fuel consumed by the combined translation-descent maneuver; the total on-board fuel requirements will also be increased.

3. There was no significant difference in fuel consumption between constant-thrust or variable-thrust engines used in ballistic maneuvers fo short translation distances (up to 20,000 ft).

4. A single-engine system requiring vehicle til for horizontal translation appears to be more efficient than a multiengine system with separate engines for translation and lift.

5. The ballistic-type translation was the mos efficient maneuver based on fuel consumption For example, fuel consumption ratio for a ballistic translation for a range of 20,000 feet was 0.05 as compared with a value of 0.065 for a constant-altitude translation of the same distance using the single-engine system.

6. For both the single-engine and multiengine systems, introducing a coast period between acceleration and retrothrust reduces the fuel consumption for a given translation.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, *August 17, 1961*

APPENDIX A

SYMBOLS

a gravity ratio, g_c/g
F thrust, lb
g acceleration due to gravity, ft/sec^2
g_c gravitational constant, 32.17 ft/sec^2
g_m lunar surface gravity, 5.4 ft/sec^2
h altitude, ft
I_{sp} specific impulse, sec
m mass, slugs
R range, ft
t time, sec
V absolute velocity, ft/sec
w local weight, lb
x distance measured parallel to x-axis, ft
$\dot{x}$ velocity measured parallel to x-axis, ft/sec
$\ddot{x}$ acceleration measured parallel to x-axis, ft/sec^2
y distance measured parallel to y-axis, ft
$\dot{y}$ velocity measured parallel to y-axis, ft/sec
$\ddot{y}$ acceleration measured parallel to y-axis, ft/sec^2
θ flight-path angle from local horizontal, deg
φ angle of thrust vector from local vertical, deg
ω rotation rate, radians/sec

Subscripts:
accel acceleration
b boost
c coast
decel deceleration
f fuel
ff free fall
h horizontal
i instantaneous
r retrothrust
rot rotation
t total
v vertical
0 initial conditions
1 end of boost
2 end of coast
3 end of free fall
4 end of retrothrust

Superscript:
— average value

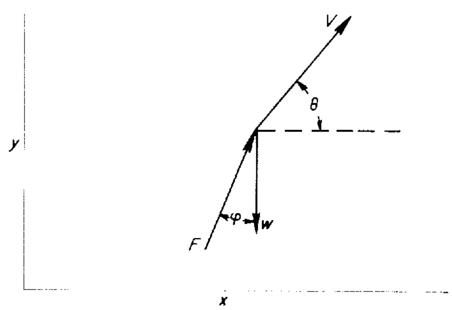
APPENDIX B

DERIVATION OF EQUATIONS

For this analysis only static trajectories of the vehicle center of gravity were considered in conjunction with the assumption of a flat nonrotating moon. The acceleration due to gravity was assumed constant for all calculations.

MANEUVER TYPES

Ballistic translation. The following sketch is presented to illustrate the coordinate system and the symbol notation used in the following derivations:



The following assumptions were made to simplify the motion equations:

(1) Constant φ during boost
(2) Magnitude of φ based on maximizing range during boost plus coast to retrorocket firing altitude (see fig. 4)
(3) Thrust vector maintained along velocity vector during retrorocket firing for constant thrust-weight-ratio case
(4) Firing angle held constant during retrorocket firing for constant-thrust case

For constant thrust-weight ratio a summation of forces in the horizontal and vertical directions during boost yields the acceleration equations:

$$\ddot{x}_b = g(F/w)\sin\varphi \qquad \text{(B1)}$$

$$\ddot{y}_b = g[(F/w)\cos\varphi - 1] \qquad \text{(B2)}$$

Integrating equations (B1) and (B2) twice gives the range and altitude equations during boost:

$$x_b = g(F/w)\sin\varphi\,\frac{t_b^2}{2} + \dot{x}_0 t_b \qquad \text{(B3)}$$

$$y_b = g[(F/w)\cos\varphi - 1]\frac{t_b^2}{2} + \dot{y}_0 t_b \qquad \text{(B4)}$$

The fuel consumption during boost is given by

$$\frac{w_f}{w_0} = 1 - exp\left[-\frac{(F_0/w_0)t_b}{aI_{sp}}\right] \qquad \text{(B5)}$$

from an integration of

$$\frac{w_i}{w_0} = 1 - \int \frac{\dot{w}}{w_0}\,dt$$

where

$$\frac{\dot{w}}{w_0} = \frac{F_0/w_0}{aI_{sp}}$$

The standard zero-drag equations were used for determining coast range and/or including a free-fall period.

The retrothrust range and altitude are given by

$$x_r = \int V\cos\theta\,dt \qquad \text{(B6)}$$

$$y_r = \int V\sin\theta\,dt \qquad \text{(B7)}$$

Equations (B6) and (B7) were solved in parametric form with an analog computer over a range of initial velocity, thrust-weight ratio, and flight-path angle to determine ignition and burning times required to obtain zero velocity at the desired altitude. The results are presented in graphical form in appendix C.

The retrothrust fuel consumption is given by

$$\frac{w_f}{w_b} = 1 - exp\left[-\frac{(F_0/w_0)t_r}{aI_{sp}}\right] \qquad \text{(B8)}$$

When constant-thrust engines are used, the acceleration equations during boost become

$$\ddot{x}_b = g\,\frac{(F_0/w_0)\sin\varphi}{1-\frac{(F_0/w_0)t}{aI_{sp}}} \tag{B9}$$

$$\ddot{y}_b = g\,\frac{(F_0/w_0)\cos\varphi}{1-\frac{(F_0/w_0)t}{aI_{sp}}}-1 \tag{B10}$$

here

$$\frac{w_i}{w_0}=1-\frac{(F_0/w_0)t}{aI_{sp}}$$

The boost range and altitude equations are by integration

$$x_b = g\,\frac{(I_{sp}a)^2}{F_0/w_0}\sin\varphi_b[A(\ln A-1)+1]+\dot{x}_0t_b \tag{B11}$$

$$y_b = g\,\frac{(I_{sp}a)^2}{F_0/w_0}\cos\varphi_b[A(\ln A-1)+1]-g\,\frac{t_b^2}{2}+\dot{y}_0t_b \tag{B12}$$

here

$$A=1-\frac{(F_0/w_0)t_b}{aI_{sp}}$$

The boost fuel consumption equation becomes simply

$$\frac{w_f}{w_0}=\frac{(F_0/w_0)t_b}{aI_{sp}} \tag{B13}$$

Zero-drag ballistic equations are used for the coast period.

The retrothrust range and altitude are given by

$$x_r = \dot{x}_rt_r+g\,\frac{I_{sp}^2a^2}{F_0/w_b}\sin\varphi_r[B(\ln B-1)+1] \tag{B14}$$

$$y_r = \dot{y}_rt_r+g\,\frac{I_{sp}^2a^2}{F_0/w_b}\cos\varphi_r[B(\ln B-1)+1]-g\,\frac{t_r^2}{2} \tag{B15}$$

here

$$B=1-\frac{(F_0/w_b)t_r}{aI_{sp}}$$

Equations (B14) and (B15) are solved by trial and error for values of φ_r and t_r until the velocity of the vehicle is zero. The retrothrust fuel consumption is then given by

$$\frac{w_f}{w_b}=\frac{(F_0/w_b)t_r}{aI_{sp}} \tag{B16}$$

Horizontal translation.—This translation is performed at constant altitude and thus requires a vertical thrust vector equal to vehicle weight at all times.

The multiengine system employs separate horizontal engines for translation purposes which maintain a constant thrust-weight ratio. The acceleration equation is then

$$\ddot{x}=g(F/w)_h \tag{B17}$$

where the subscript h denotes horizontal engine. A double integration of equation (B17) yields the range equation

$$x_b=\frac{1}{2}\,g(F/w)_ht_b^2 \tag{B18}$$

Since an equal time is spent during deceleration of the vehicle, the total range equation becomes $x_t=g(F/w)_ht_b^2$ for acceleration and deceleration. If a coast period is introduced between acceleration and deceleration, the total range equation becomes

$$x_t=g(F/w)_ht_b^2+g(F/w)_ht_bt_c \tag{B18a}$$

where t_c is the time of coast.

The total fuel consumed during translation including the coast period is as follows:

$$\frac{w_f}{w_0}=1-exp\left\{-\frac{2[1+(F/w)_h]t_b}{aI_{sp}}-\frac{t_c}{aI_{sp}}\right\} \tag{B19}$$

By substituting for t_b from equation (B18a) into (B19), the fuel consumption equation becomes

$$\frac{w_f}{w_0}=1-exp\left\{-\frac{2}{aI_{sp}}\,[1+(F/w)_h]\left[-\frac{t_c}{2}+\sqrt{\frac{t_c^2}{4}+\frac{x_t}{g(F/w)_h}}\right]-\frac{t_c}{aI_{sp}}\right\} \tag{B20}$$

The total fuel consumption is a function of three variables for a selected propellant, namely, $(F/w)_h$, t_c, and x_t. In order to minimize the fuel consumption, the absolute value of the exponent of equation (B20) must be minimized. For a given engine size, $(F/w)_h$, and range, the optimum coast time is obtained by taking the partial derivative of the exponent of equation (B20) with respect to t_c and equating it to zero; the resulting optimum is

$$t_c=2\sqrt{\frac{x_t}{g}}\sqrt{\frac{F/w}{1+2(F/w)}} \tag{B21}$$

Substituting equation (B15) back into (B20) gives the minimum fuel consumption for a given $(F/w)_h$ and x_t:

$$\frac{w_f}{w_0}=1-exp\left[-\frac{2}{aI_{sp}}\sqrt{\frac{x_t}{g}}\sqrt{\frac{2(F/w)+1}{F/w}}\right] \quad \text{(B22)}$$

For a single-engine system that requires vehicle rotation to provide horizontal acceleration, the equation for horizontal acceleration during rotation is

$$\ddot{x}=g \tan \omega t \quad \text{(B23)}$$

where ω is an assumed constant rotation rate. The approximate range equation resulting from a double integration of equation (B23) is

$$x_{rot}=\frac{g}{\omega^2}\left(\frac{\varphi_h^3}{6}+\frac{\varphi_h^5}{60}+\frac{\varphi_h^7}{315}\right) \quad \text{(B24)}$$

where, in the second integration, the ln cos φ was approximated by the series $-\frac{\varphi^2}{2}-\frac{\varphi^4}{12}-\frac{\varphi^6}{45}$. The horizontal acceleration following rotation is given by

$$\ddot{x}_h=g(F/w)\sin\varphi_h \quad \text{(B25)}$$

where φ_h is in radians and is held constant during translation. The total range equation including range during rotation, coast, acceleration, and deceleration is given by

$$x_t=4\frac{g}{\omega^2}\left(\frac{\varphi_h^3}{6}+\frac{\varphi_h^5}{60}+\frac{\varphi_h^7}{315}\right)-\frac{4g}{\omega}\ln\cos\varphi_h\left(\frac{\varphi_h}{\omega}+\frac{t_b}{2}\right)$$
$$+g(F/w)\sin\varphi_h\left(t_b^2+2t_b\frac{\varphi_h}{\omega}\right)$$
$$+g(F/w)\sin\varphi_h t_c t_b-\frac{gt_c}{\omega}\ln\cos\varphi_h \quad \text{(B26)}$$

The total fuel consumption (equation excludes fuel required to torque vehicle) is given by

$$\left(\frac{w_f}{w_0}\right)_t=1-\left[(\sec\varphi_h+\tan\varphi_h)^{-\frac{4}{\omega a I_{sp}}}\right]$$
$$exp\left[-\frac{2(F/w)t_b}{aI_{sp}}-\frac{t_c}{aI_{sp}}\right] \quad \text{(B27)}$$

In the case where vehicle rotation is neglected, equation (B27) reduces to

$$\frac{w_f}{w_0}=1-exp\left[-\frac{2(F/w)t_b}{aI_{sp}}-\frac{t_c}{aI_{sp}}\right] \quad \text{(B28)}$$

From equation (B26) (neglecting vehicle rotatio the equation for t_b becomes

$$t_b=-\frac{t_c}{2}+\sqrt{\frac{t_c^2}{4}+\frac{x_t}{g(F/w)\sin\varphi}} \quad \text{(B2}$$

Substituting equation (B29) into (B28) yields t fuel consumption equation

$$\frac{w_f}{w_0}=1-exp\left\{-\frac{2(F/w)}{aI_{sp}}\right.$$
$$\left.\left[-\frac{t_c}{2}+\sqrt{\frac{t_c^2}{4}+\frac{x_t}{g(F/w)\sin\varphi}}\right]-\frac{t_c}{aI_{sp}}\right\} \quad \text{(B3}$$

From the condition of constant-altitude transla tion, it is evident that

$$(F/w)\cos\varphi=1$$

Then

$$\sin\varphi=\frac{\sqrt{(F/w)^2-1}}{(F/w)} \quad \text{(B3}$$

Substituting equation (B31) into (B30) results i the fuel equation

$$\frac{w_f}{w_0}=1-exp\left(-\frac{t_c}{aI_{sp}}\left\{\sqrt{(F/w)^2+\frac{4x_t(F/w)^2}{gt_c^2[(F/w)^2-1]^{1/2}}}\right.\right.$$
$$\left.\left.-[(F/w)-1]\right\}\right) \quad \text{(B3}$$

For minimum fuel consumption, the exponent i equation (B32) must be minimized by use of parti derivatives. In the case of a given engine siz F/w, and range, the optimum coast time is

$$t_c=2\sqrt{\frac{x_t}{g}}\left\{\frac{(F/w)-1}{\sqrt{[2(F/w)-1][(F/w)^2-1]^{1/2}}}\right\} \quad \text{(B3}$$

Substituting equation (B33) into (B32) and r arranging yield

$$\frac{w_f}{w_0}=1-exp\left\{-\frac{2}{aI_{sp}}\sqrt{\frac{x_t}{g}}\sqrt{\frac{2(F/w)-1}{[(F/w)^2-1]^{1/2}}}\right\} \quad \text{(B3}$$

which is the equation for minimum fuel co sumption for translation with a single-engi system neglecting vehicle rotation.

APPENDIX C

CHART SOLUTIONS OF BALLISTIC EQUATIONS

Charts are presented in this appendix as an aid in obtaining preliminary numbers for system requirements for ballistic translational maneuvers. The charts are useful because an iteration process is necessary to obtain the solution of ballistic motion equations in this application.

The following is a list of the charts presented indicating the equation solved through the use of the chart (see fig. 15 for the symbol notation):

Chart I. Represents the solution of the equation

$$\Delta\dot{y}_b = g[(F/w)\cos\varphi - 1]t_b$$

for optimum firing angle φ (constant for given F/w, as discussed in appendix B). $\Delta\dot{y}_b$ is the vertical component of velocity at the end of applied thrust.

Chart II. Represents the solution of the equation

$$\Delta h_b = \frac{g}{2}\left(\frac{F}{w}\cos\varphi - 1\right)t_b^2$$

Δh_b is the altitude change due to applied thrust.

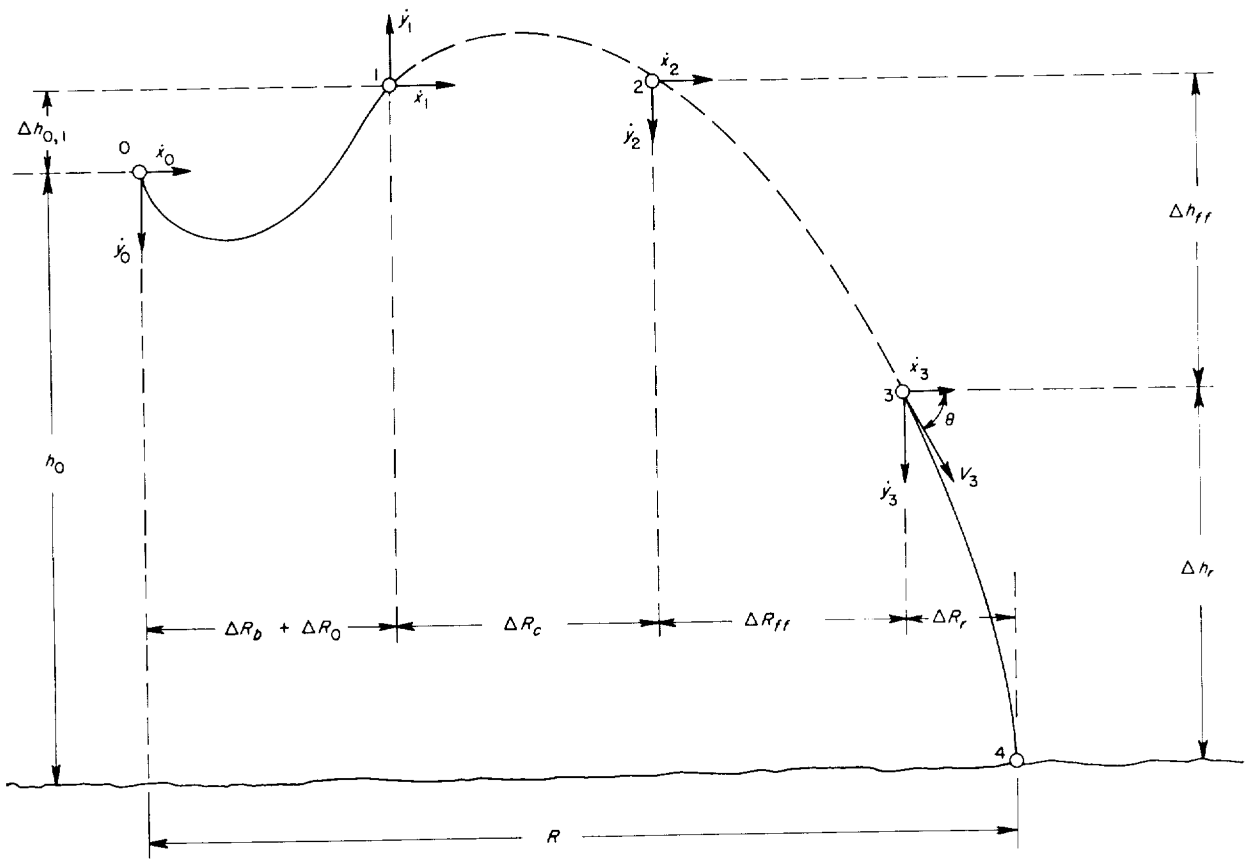


FIGURE 15.—Typical ballistic trajectory.

CHART I.—Variation of thrust-weight ratio with vertical burnout velocity.

CHART II.—Variation of change in altitude with vertical burnout velocity.

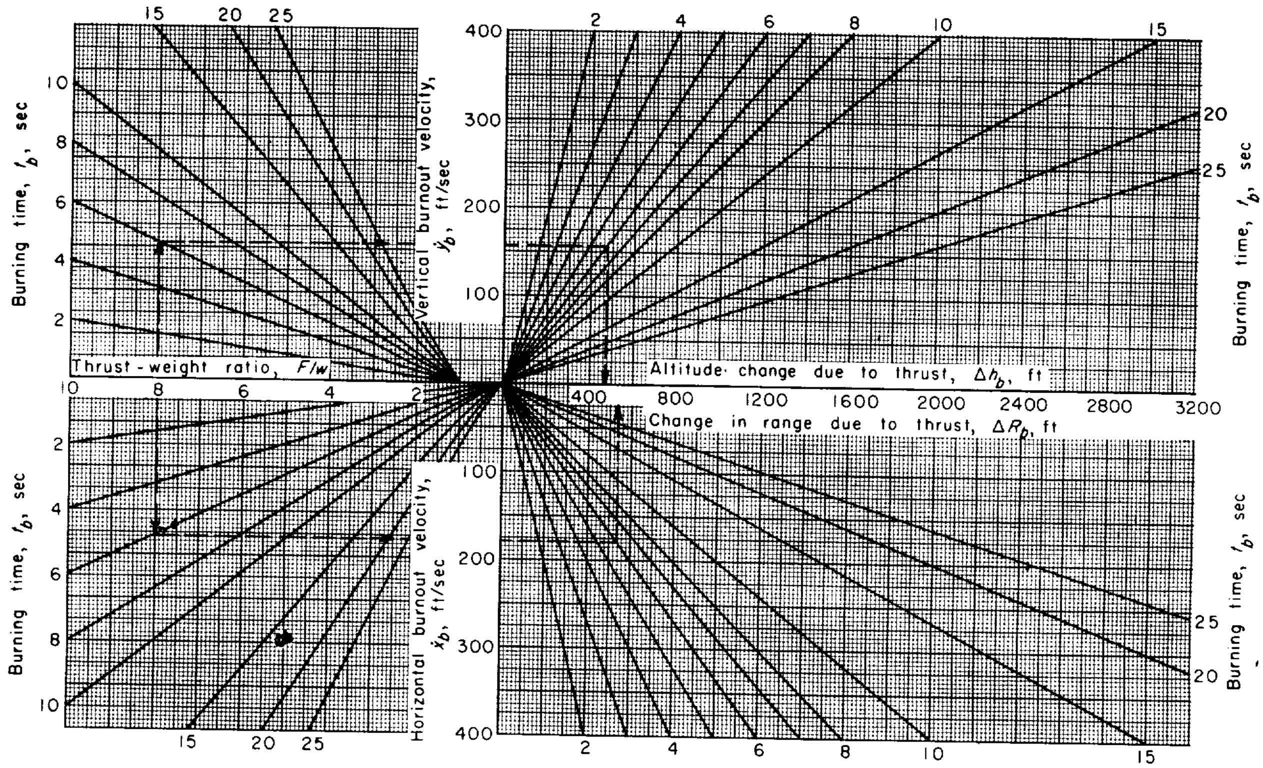


CHART III.—Variation of thrust-weight ratio with horizontal burnout velocity.

CHART IV. Variation of change in range with horizontal burnout velocity.

Chart III. Represents the solution of the equation

$$\dot{x}_b = g\frac{F}{w}\sin\varphi t_b$$

$\dot{x}_b$ is the horizontal component of velocity due to applied thrust.

Chart IV. Represents the solution of the equation

$$\Delta R_b = \frac{g}{2}\frac{F}{w}\sin\varphi t_b^2$$

ΔR_b is the range change due to applied thrust.

Chart V. Gives the final vertical velocity component after burnout

$$\dot{y}_1 = \dot{y}_0 + \Delta\dot{y}_b$$

Chart VI. Represents the altitude or range change due to initial vertical or horizontal velocity

$$\Delta h_0 = \dot{y}_0 t_b \text{ or } \Delta R_0 = \dot{x}_0 t_b$$

The total altitude change after burnout is then the sum of Δh_b from chart II and Δh_0, and the total range change is the sum of ΔR_b from chart IV and ΔR_0 from chart VI without the minus sign.

Chart VII. Represents the solution of the equation

$$t_c = 2\dot{y}_1$$

t_c is the coast time required to return to burnout altitude $h_1 = h_2$ if $\dot{y}_1$ is positive (upward).

CHART V.—Final vertical velocity component after burnout.

CHART VI.—Variation of change in altitude with initial vertical velocity.

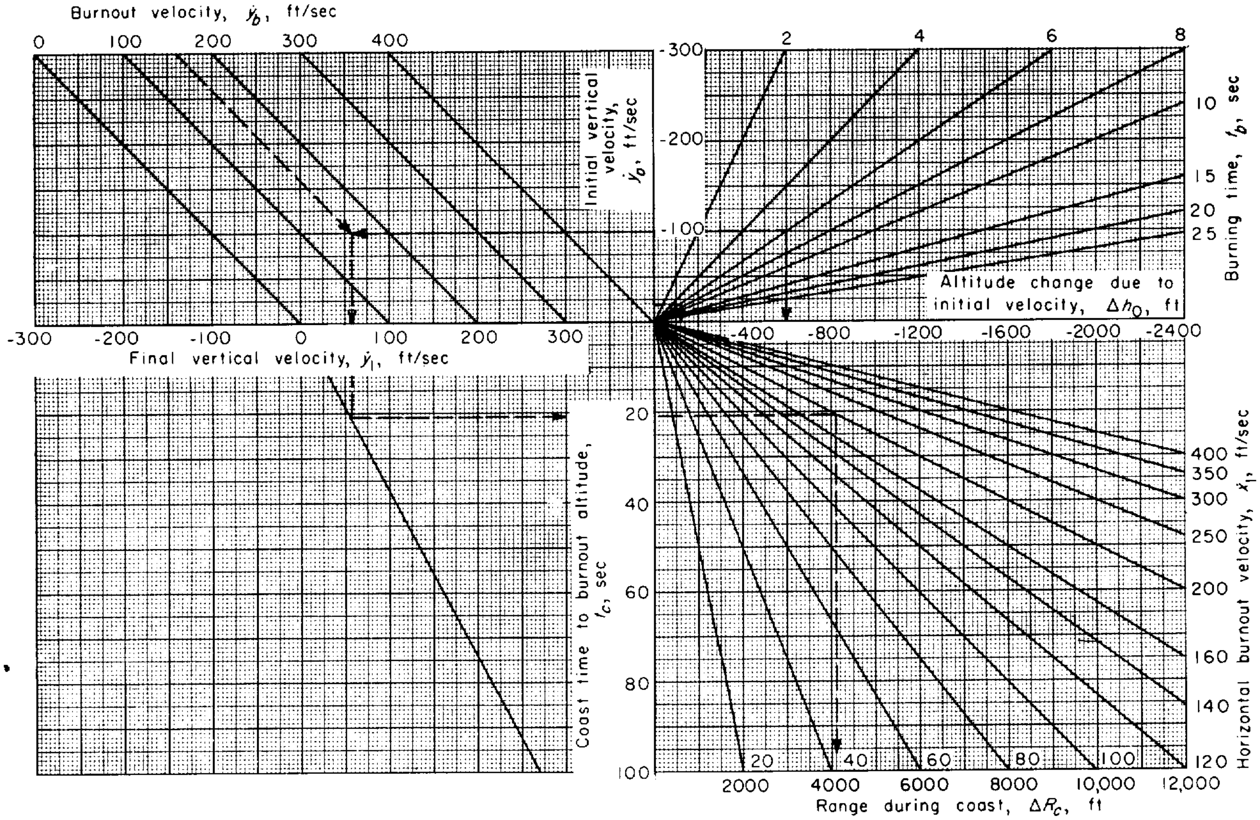


CHART VII. Variation of vertical velocity with coast time to burnout altitude.

CHART VIII. Variation of change in range during coast with coast time to burnout altitude.

Chart VIII. Represents the change in range due to the coast period

$$\Delta R_c = \dot{x}_1 t_c = \dot{x}_b t_c$$

Chart IX. Represents the altitude change during free fall due to vertical velocity and free-fall time

$$\Delta h_{ff} = \dot{y}_2 t_{ff} + g \frac{t_{ff}^2}{2}$$

Chart X. Represents the solution of the equation

$$\dot{y}_3 = \sqrt{\dot{y}_2^2 + 2g\Delta h_{ff}}$$

where $\dot{y}_3$ is the vertical velocity after free fall.

Chart XI. Represents the change in range during free fall given by the equation

$$\Delta R_{ff} = \dot{x}_3 t_{ff}$$

Chart XII. Represents the relation between the absolute velocity vector and its horizontal and vertical components as given by

$$V_3 = \frac{\dot{x}_3}{\cos\theta} = \frac{\dot{y}_3}{\sin\theta}$$

where θ is the flight-path angle.

Chart XIII. Represents the analog solution of the equation of motion during retrothrust application (along the velocity vector) for $F/w = 2$, 4, 6, 8, and 10. The changes in range and altitude are given by

$$\Delta R_r = \int_{t=0}^{t} V_3 \cos\theta \, dt$$

$$\Delta h_r = \int_{t=0}^{t} V_3 \sin\theta \, dt$$

for $V_4 = 0$ and $\theta = 90°$.

CHART IX.—Altitude change during free fall due to vertical velocity and free-fall time.

CHART X.—Variation of vertical velocity after free fall with free-fall altitude.

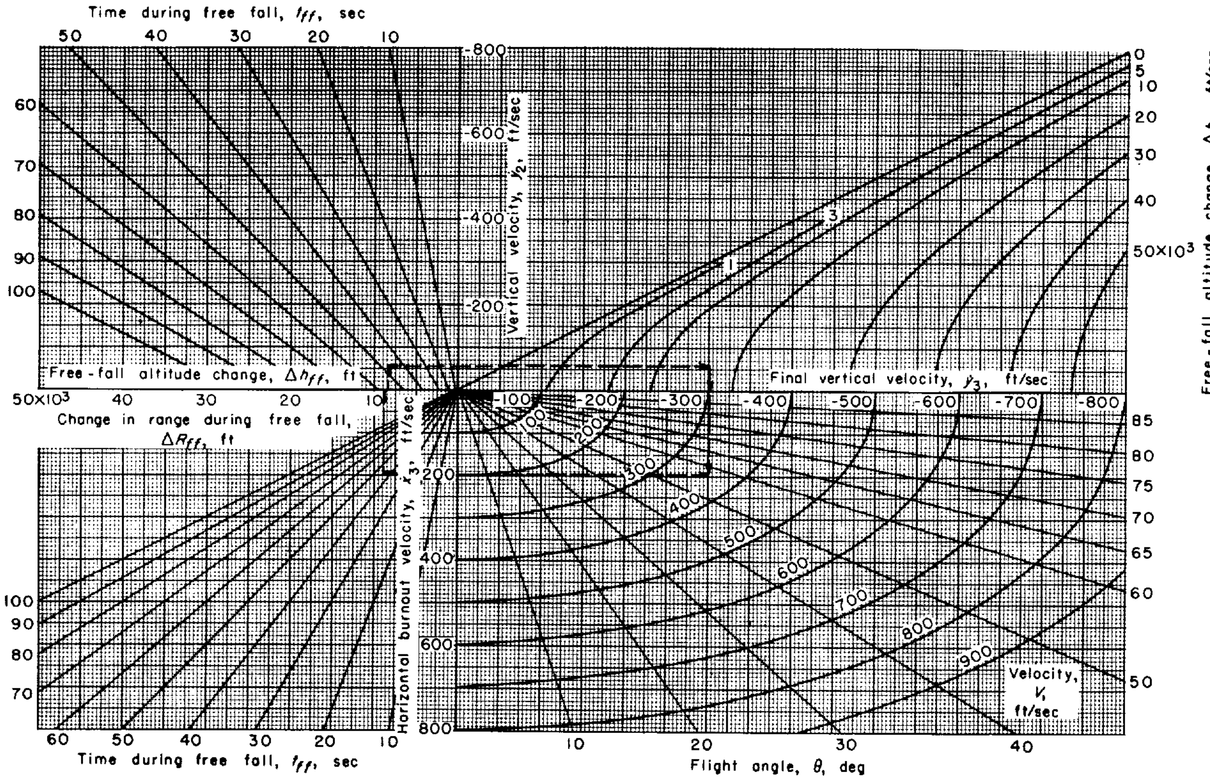


CHART XI. Change in range during free fall.

CHART XII.—Relation between absolute velocity vector and horizontal and vertical components.

To illustrate the use of the included charts for the solution of the ballistic translational maneuver, step-by-step solutions of two sample problems are presented herein.

Example 1: Determine the fuel consumption ratio for a horizontal translation of approximately 3 miles from an initial altitude of 10,000 feet. The initial horizontal and vertical velocities are 20 and −100 feet per second (downward velocity), respectively.

A thrust-weight ratio of 8 is specified with the first trial boost burning time t_b of 6 seconds selected.

(1) Determine $\dot{y}_b$ and $\dot{x}_b$.
From chart I, $F/w=8$, $t_b=6$ sec
$\dot{y}_b=157$ ft/sec
From chart III, $F/w=8$, $t_b=6$ sec
$\dot{x}_b=178$ ft/sec

(2) Determine altitude and range change due to burning.
From chart II, $\dot{y}_b=157$, $t_b=6$ sec
$\Delta h_b=475$ ft
From chart IV, $\dot{x}_b=178$, $t_b=6$ sec
$\Delta R_b=535$ ft

(3) Calculate $\dot{y}_1$ and $\dot{x}_1$.
From chart V, $\dot{y}_1=57$ ft/sec
$\dot{x}_1=\dot{x}_0+\dot{x}_b=20+178=198$ ft/sec

(4) Determination of ΔR_0 and Δh_0.
From chart VI, $\dot{x}_0=20$ ft/sec,
$y_0=-100$ ft/sec
$\Delta h_0=-600$ ft
and, neglecting the minus signs for the range case,
$\Delta R_0=120$ ft

(5) Determine the change in range during coast return to burnout altitude from charts VII and VIII and $\dot{x}_1=198$ ft/sec.
$t_c=21$ sec
$\Delta R_c=4100$ ft

(6) Determine the change in range during free fall and absolute velocity and flight-path angle prior to retrothrust.
Select free-fall altitude.
Trial 1. $\Delta h_{ff}=8150$ ft
From $\dot{y}_2=-\dot{y}_1=-57$ ft/sec (step (3)) and chart IX
$t_{ff}=45$ sec
From chart X, $\dot{y}_3=-302$ ft/sec
From chart XI, $\dot{x}_3=\dot{x}_1=198$ ft/sec (step (3))
$\Delta R_{ff}=8900$ ft
From chart XII, $\dot{y}_3=-302$ ft/sec, $\dot{x}_3=198$ ft/sec
$v_3=367$ ft/sec, $\theta=57°$

(7) Determine change in range and altitude during retrothrust.
From chart XIII, $v_3=367$ ft/sec, $\theta=57°$ (step (2))
$\Delta R_r=900$ ft
$\Delta h_r=1500$ ft

(8) Summation of range and altitude.
$R=\Delta R_0+\Delta R_b+\Delta R_c+\Delta R_{ff}+\Delta R_r$
$=120+535+4100+8900+900$
$=14{,}555$ ft
$\Delta h=\Delta h_0+\Delta h_b+\Delta h_{ff}+\Delta h_r$
$=-600+475-8150-1500$
$=-9775$ ft
Repeat steps (6) to (8) with $\Delta h_{ff}=8350$ ft
$\Delta h_{ff}=8350$ ft
Step (6a)
$t_{ff}=47$ sec
$\dot{y}_3=-305$ ft/sec
$\Delta R_{ff}=9300$ ft
$v_3=370$ ft/sec, $\theta=57°$
Step (7a)
$v_3=370$ ft/sec
$\Delta R_r=920$ ft
$\Delta h_r=1520$ ft
Step (8a)
$R=14{,}975$ ft
$\Delta h=-9995$ ft

(9) Determination of theoretical fuel consumption.
From chart XIII(d), $t_r=9.5$ sec, $t_b=6$ sec
Total burning time
$t_t=t_r+t_b=15.5$ sec

$$\frac{w_f}{w_0}=1-exp\left[-\frac{(F/w)\,t_t}{aI_{sp}}\right]$$
$$=1-exp\left(-\frac{8\times15.5}{400}\times\frac{5.4}{32.17}\right)$$
$$=1-exp\,(-0.052)=1-0.949=0.051$$

$$\frac{w_f}{w_0}=5.1 \text{ percent}$$

Example 2: Determine the fuel consumption ratio and range for the initial conditions of example 1 when the vertical velocity component is increased from -100 to -200 feet per second. This will serve to illustrate a ballistic translation where no coast period occurs after boost burnout.

(1) Determine $\dot{y}_b$ and $\dot{x}_b$.
From chart I, $F/w=8$, $t_b=6$ sec
$\dot{y}_b=157$ ft/sec
From chart III, $F/w=8$, $t_b=6$ sec
$\dot{x}_b=178$ ft/sec

(2) Determine altitude and range change due to burning.
From chart II, $\dot{y}_b=157$ ft/sec, $t_b=6$ sec
$\Delta h_b=475$ ft
From chart IV, $\dot{x}_b=178$ ft/sec, $t_b=6$ sec
$\Delta R_b=535$ ft

(3) Calculate $\dot{y}_1$ and $\dot{x}_1$.
From chart V, $\dot{y}_1=-43$ ft/sec
$\dot{x}_1=\dot{x}_0+\dot{x}_b=20+178=198$ ft/sec

(4) Determination of ΔR_0 and Δh_0.
From chart VI, $\dot{x}_0=20$ ft/sec and $\dot{y}_0=-200$ ft/sec
$\Delta h_0=-1200$ ft
and, neglecting the minus signs for the range case,
$\Delta R_0=120$ ft

(5) Inasmuch as $\dot{y}_1$ is downward (-43 ft/sec from chart V), there is no coast period as in example 1 and free fall occurs immediately after the boost phase.

(6) Determine the change in range during free fall and absolute velocity and flight-path angle prior to retrothrust. Select free-fall altitude.
Trial 1. $\Delta h_{ff}=8000$ ft
From $\dot{y}_1=-43$ ft/sec (step (3)) and chart IX, $t_{ff}=47.0$ sec
From chart X, $\dot{y}_3=297$ ft/sec

From chart XI, $\dot{x}_3=\dot{x}_1=198$ ft/sec (step (3)), $\Delta R_{ff}=9315$ ft
From chart XII, $\dot{y}_3=-297$ ft/sec, $\dot{x}_3=198$ ft/sec
$V_3=357$ ft/sec, $\theta=56.3°$

(7) Determine change in range and altitude during retrothrust.
From chart XIII, $V_3=357$ ft/sec, $\theta=56.3°$ (step (6))
$\Delta R_r=860$ ft
$\Delta h_r=1430$ ft

(8) Summation of range and altitude.
$R=\Delta R_0+\Delta R_b+\Delta R_c+\Delta R_{ff}+\Delta R_f$
$=120+535+0+9315+860$
$=10{,}830$ ft
$\Delta h=\Delta h_0+\Delta h_b+\Delta h_{ff}+\Delta h_r$
$=-1200+475-8000-1430$
$=-10{,}155$ ft
Repeat steps (6) to (8) with $\Delta h_{ff}=7850$ ft.
Step (6a)
$t_{ff}=46.5$ sec
$\dot{y}_3=294$ ft/sec
$\Delta R_{ff}=9210$ ft
$V_3=353$ ft/sec, $\theta=56°$
Step (7a)
$V_3=355$ ft/sec
$\Delta R_r=870$ ft
$\Delta h_r=1400$ ft
Step (8a)
$R=10{,}735$ ft
$\Delta h=9975$ ft

(9) Determination of theoretical fuel consumption.
From chart XIII(d), $t_r=9.14$ sec, $t_b=6$ sec
Total burning time $t_t=t_r+t_b=15.14$ sec

$$\frac{w_f}{w_0}=1-exp\left[-\frac{(F/w)t_t}{aI_{sp}}\right]$$
$$=1-exp\left[-\frac{8(15.14)}{400}\times\frac{5.4}{32.17}\right]$$
$$=1-exp(-0.0508)=1-0.95=0.050$$

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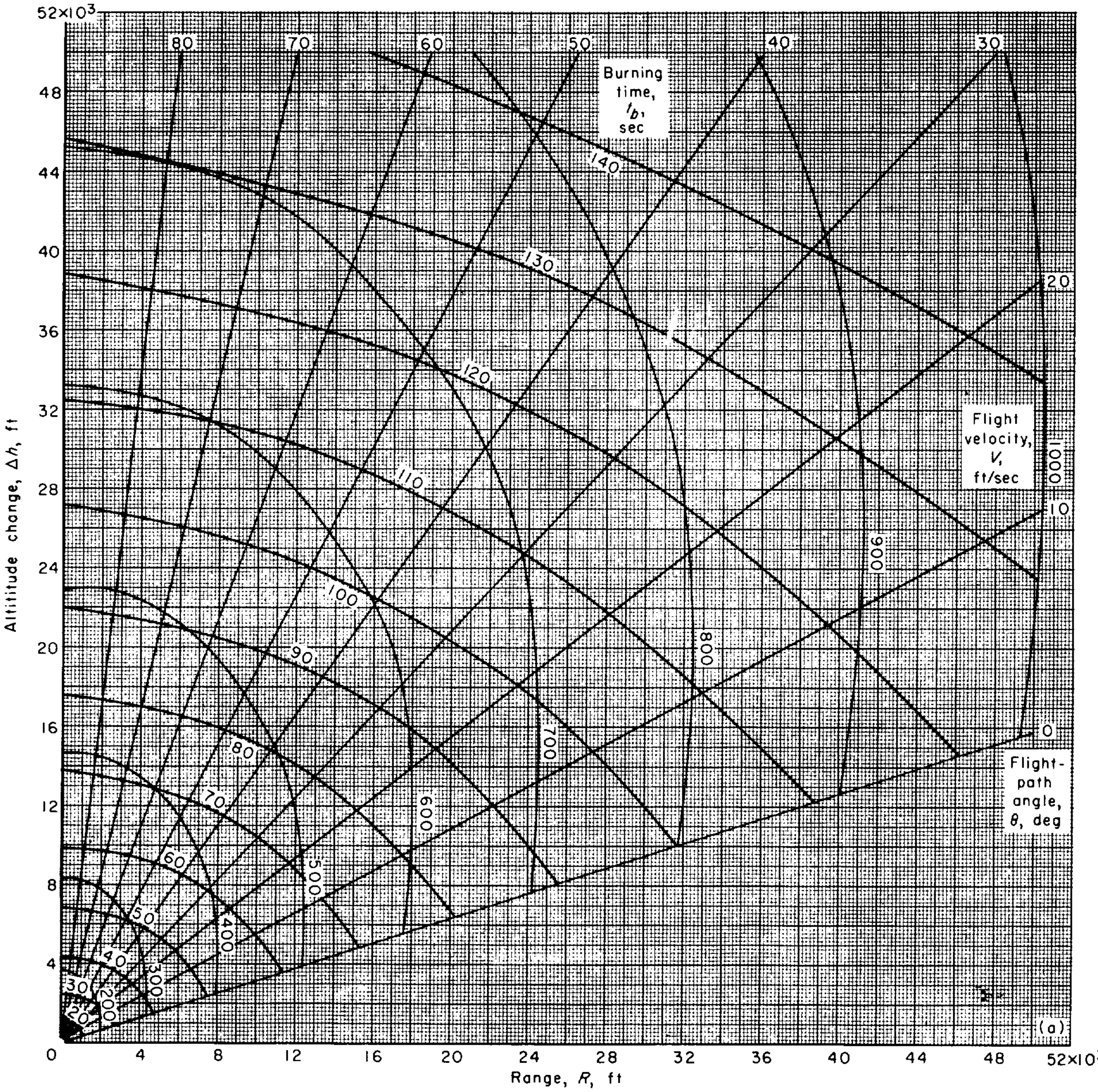
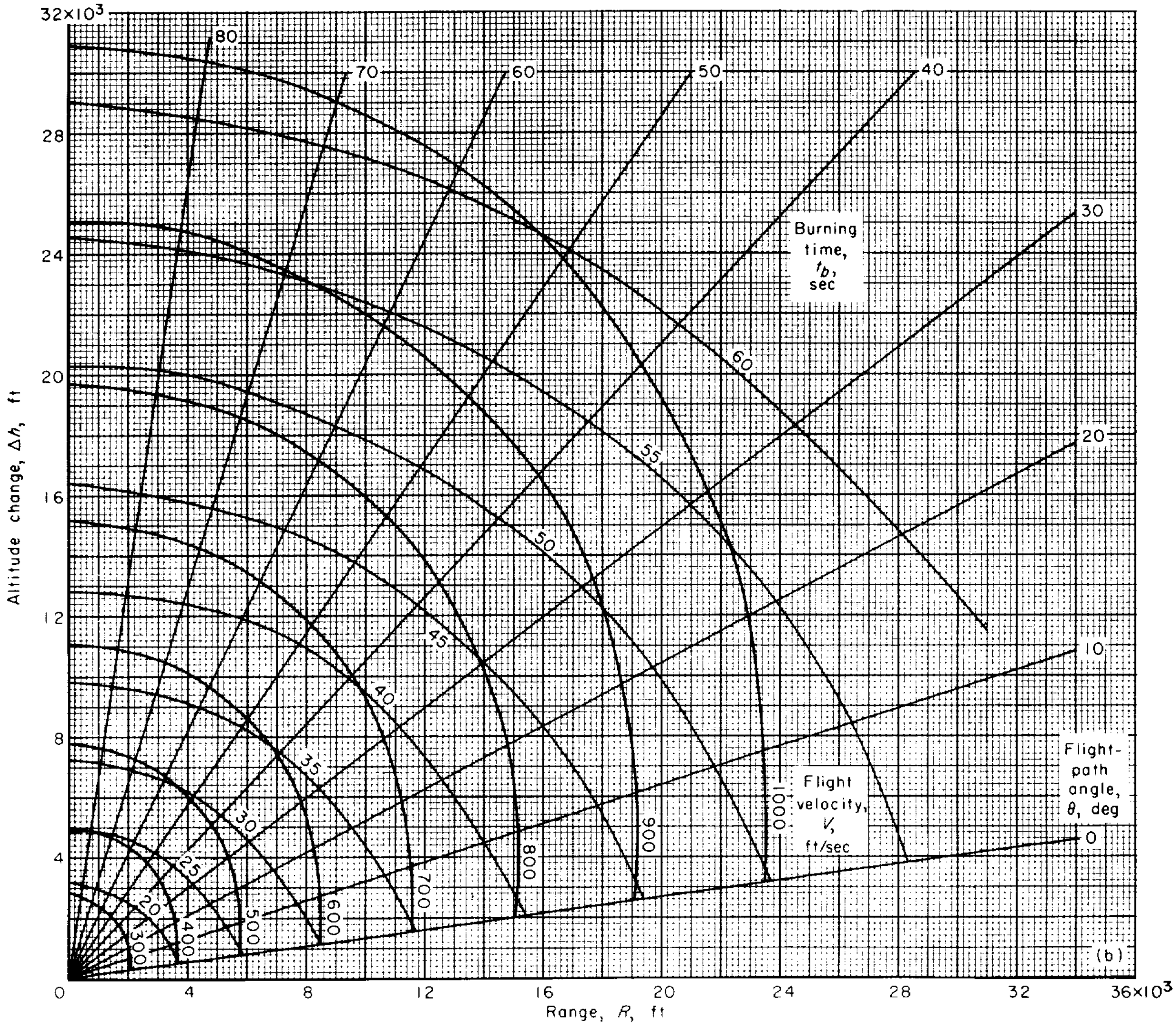
1. Weber, Richard J., and Pauson, Werner M.: Some Thrust and Trajectory Considerations for Lunar Landings. NASA TN D-134, 1959.
2. Wrobel, J. Richard, and Breshears, Robert R.: Lunar Landing Vehicle Propulsion Requirements. Tech. Release 34-66, Jet Prop. Lab., C.I.T., May 1, 1960.



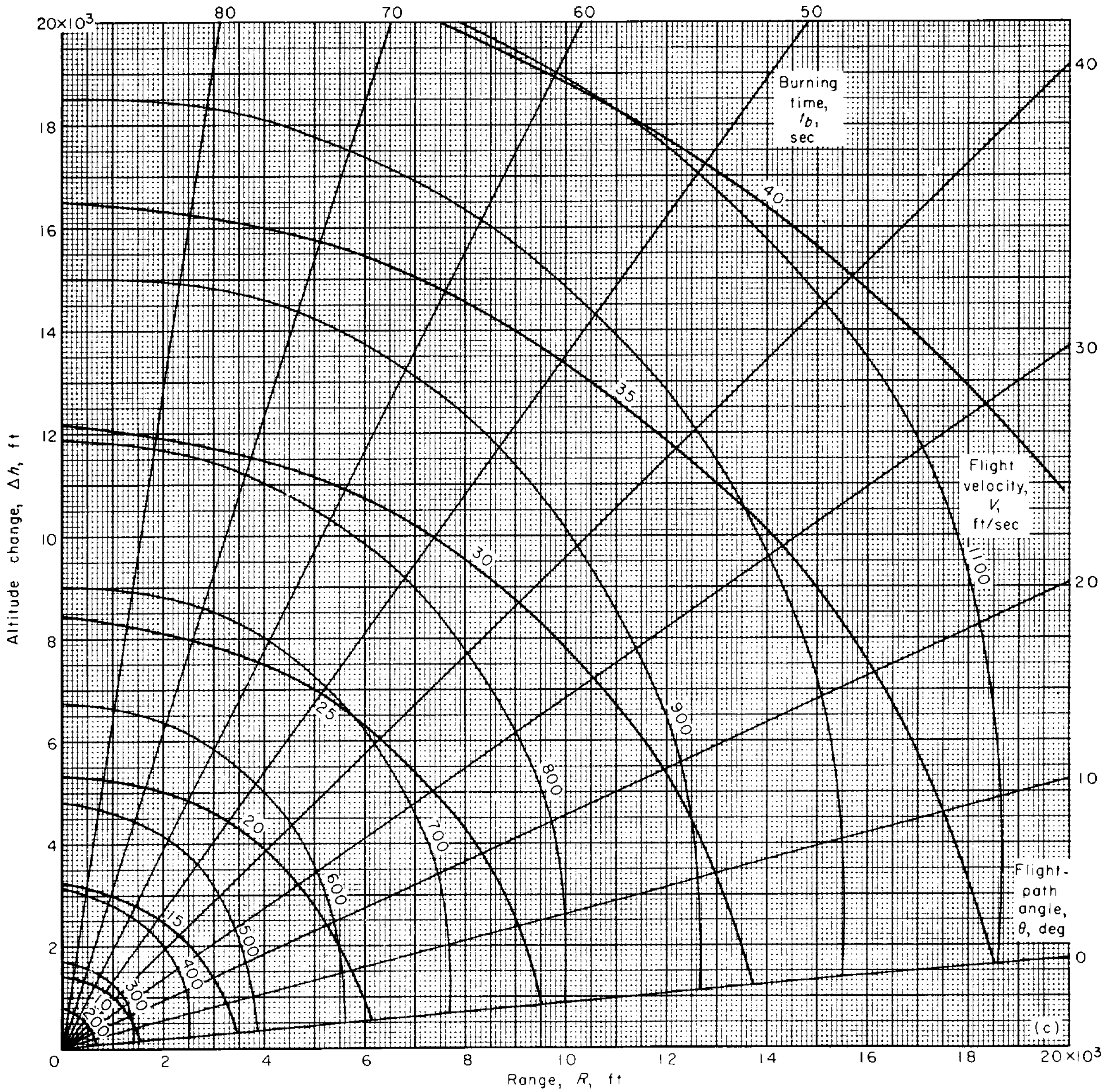
(a) Thrust-weight ratio, 2.

CHART XIII. Altitude change, range, and burning time during final retrothrust as functions of initial flight velocity and flight-path angle.



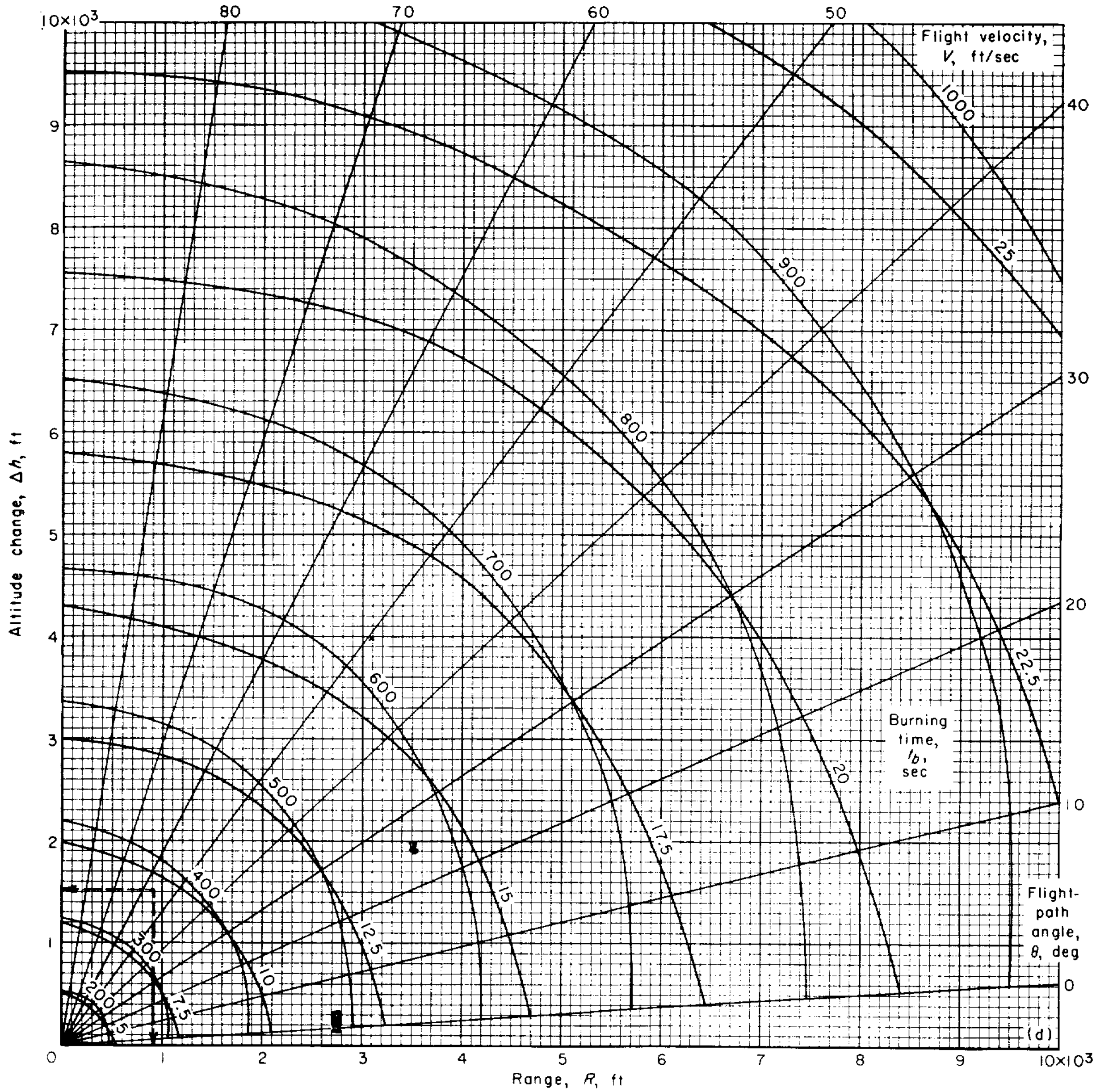
(b) Thrust-weight ratio, 4.

CHART XIII. Continued. Altitude change, range, and burning time during final retrothrust as functions of initial flight velocity and flight-path angle.



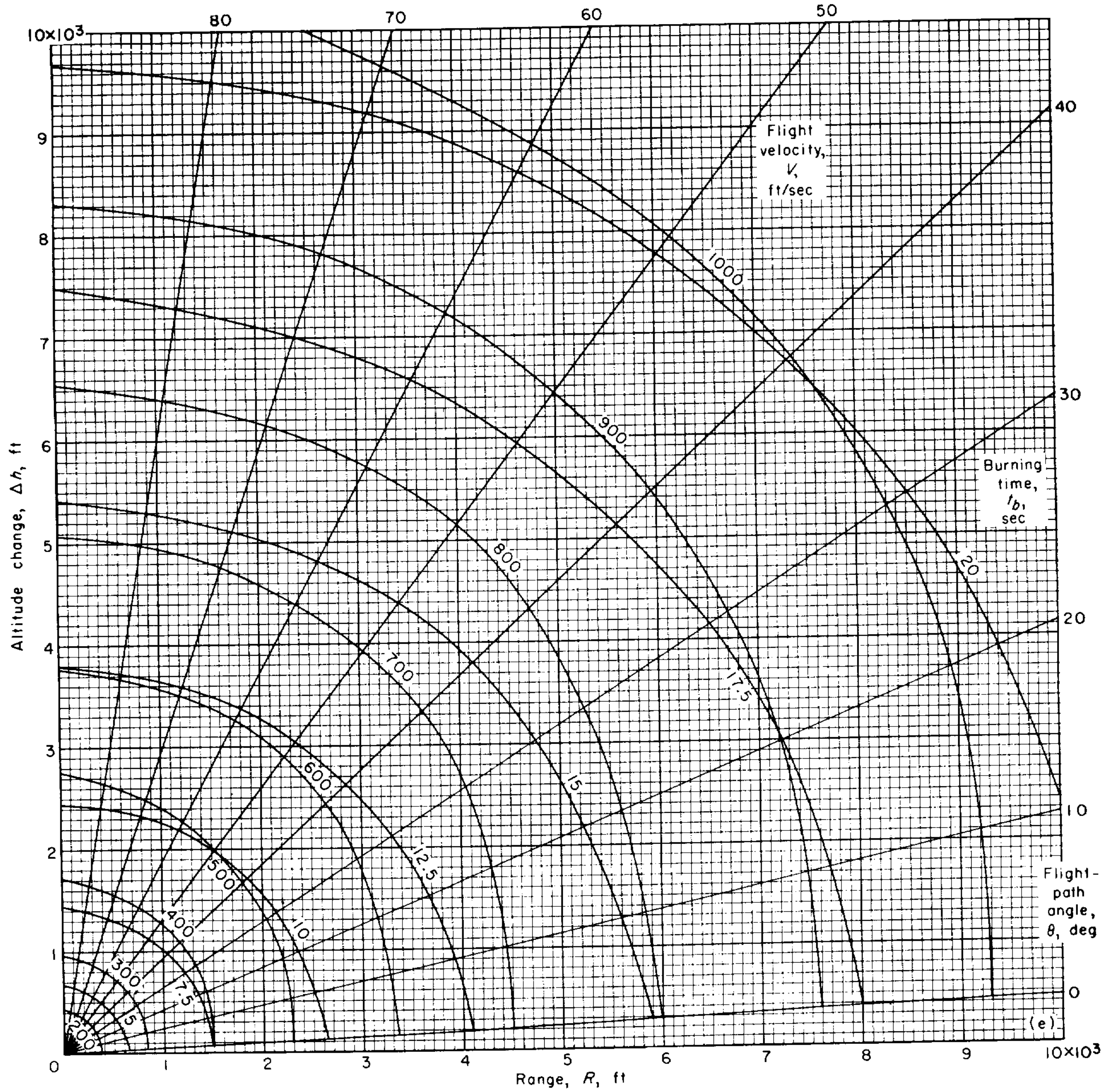
(c) Thrust-weight ratio, 6.

CHART XIII. Continued. Altitude change, range, and burning time during final retrothrust as functions of initial flight velocity and flight-path angle.



(d) Thrust-weight ratio, 8.

CHART XIII. Continued. Altitude change, range, and burning time during final retrothrust as functions of initial flight velocity and flight-path angle.



(e) Thrust-weight ratio, 10.

CHART XIII.—Concluded. Altitude change, range, and burning time during final retrothrust as functions of initial flight velocity and flight-path angle.

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